

Difference equations everywhere: some motivating examples

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Abstract This work collects several situations where discrete dynamical systems or difference equations appear. Most of them are different from the examples used in textbooks and from the usual mathematical models appearing in Biology or Economy. The examples are presented in detail, including some appropriate references. Although most of them are known, the fact of collecting all together aims to be a source of motivation for studying DDS and difference equations and to facilitate teaching these subjects.

1 Introduction

The main goal of the theory of discrete dynamical systems (DDS) is to study the limit behavior of the sequence $\{x_n\}_n$, defined iteratively as $x_{n+1} = F(x_n)$, in terms of the initial condition x_0 , where F is an invertible map defined on a given space. When F is not invertible sometimes it is said that it defines a semi-DDS. In particular, many difference equations and recurrences can be interpreted as semi-DDS or DDS.

They also appear frequently in problems of other branches of Mathematics. Without aiming to be exhaustive, some examples are: the iterative methods proposed to approximate the solutions of linear or non-linear systems, the Bernoulli iterative method to find the dominant root of a polynomial, the numerical schemes like the Euler, Taylor or Runge-Kutta methods designed to approximate the solutions of ordinary differential equations, the schemes of differences used to approximate the solutions of partial differential equations, the study of discrete Markov chains, the complex dynamics, ...

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