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Traveling surface waves of moderate amplitude in shallow water

Armengol Gasull, Anna Geyer *

Departament de Matemàtiques, Facultat de Ciències, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

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ABSTRACT

We study traveling wave solutions of an equation for surface waves of moderate amplitude arising as a shallow water approximation of the Euler equations for inviscid, incompressible and homogeneous fluids. We obtain solitary waves of elevation and depression, including a family of solitary waves with compact support, where the amplitude may increase or decrease with respect to the wave speed. Our approach is based on techniques from dynamical systems and relies on a reformulation of the evolution equation as an autonomous Hamiltonian system which facilitates an explicit expression for bounded orbits in the phase plane to establish existence of the corresponding periodic and solitary traveling wave solutions.

1. Introduction and main result

A number of competing nonlinear model equations for water waves have been proposed to this day to account for fascinating phenomena, such as wave breaking or solitary waves, which are not captured by linear theory. The well-known Camassa–Holm equation [1] is one of the most prominent examples, due to its rich structural properties. It is an integrable infinite-dimensional Hamiltonian system [2–4] whose solitary waves are solitons [5,6]. Some of its classical solutions develop singularities in finite time in the form of wave breaking [7], and recover in the sense of global weak solutions after blow up [8,9]. For a discussion on integrability in the periodic case we refer the reader to [10,11], and a classification of weak traveling wave solutions of the Camassa–Holm equation may be found in [12]. The manifold of its enticing features led Johnson to demonstrate the relevance of the Camassa–Holm equation as a model for the propagation of shallow water waves of moderate amplitude. He proved that the horizontal component of the fluid velocity field at a certain depth within the fluid is indeed described by a Camassa–Holm equation [13,14]. Constantin and Lannes [15] followed up on the matter in search of a suitable corresponding equation for the free surface and derived an evolution equation for surface waves of moderate amplitude in the shallow water regime,

$$u_t + u_x + 6uu_x - 6u^2u_x + 12u^3u_x + u_{xxx} - u_{xxt} + 14uu_{xxx} + 28u_xu_{xx} = 0. \tag{1}$$

The authors show that Eq. (1) approximates the governing equations to the same order as the Camassa–Holm equation, and also prove that the Cauchy problem on the line associated to (1), is locally well-posed [15]. Employing a semigroup approach due to Kato [16], Duruk [17] shows that this result also holds true for a larger class of initial data, as well as for the corresponding spatially periodic Cauchy problem [18]. Consequently, solutions of (1) depend continuously on their initial data in H^s for $s > 3/2$, and it can be shown that this dependence is not uniformly continuous [19]. In the context of Besov

* Corresponding author. Tel.: +34 639256459.
E-mail addresses: gasull@mat.uab.cat (A. Gasull), anna.geyer@univie.ac.at, annageyer@mat.uab.cat, geyer.anna@gmail.com (A. Geyer).

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