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J. Differential Equations 261 (2016) 2141–2167

Journal of Differential Equations

www.elsevier.com/locate/jde

On the number of limit cycles for perturbed pendulum equations

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Received 1 February 2016; revised 20 April 2016

Available online 4 May 2016

Abstract

We consider perturbed pendulum-like equations on the cylinder of the form $\ddot{x} + \sin(x) = \varepsilon \sum_{s=0}^{m} Q_{n,s}(x)\,\dot{x}^s$ where $Q_{n,s}$ are trigonometric polynomials of degree n, and study the number of limit cycles that bifurcate from the periodic orbits of the unperturbed case $\varepsilon = 0$ in terms of m and n. Our first result gives upper bounds on the number of zeros of its associated first order Melnikov function, in both the oscillatory and the rotary regions. These upper bounds are obtained expressing the corresponding Abelian integrals in terms of polynomials and the complete elliptic functions of first and second kind. Some further results give sharp bounds on the number of zeros of these integrals by identifying subfamilies which are shown to be Chebyshev systems.

MSC: primary 34C08; secondary 34C07, 34C25Â, 37G15, 70K05

Keywords: Limit cycles; Perturbed pendulum equation; Infinitesimal Sixteenth Hilbert problem; Abelian integrals

1. Introduction

The so-called *Hilbert's 16th Problem* was proposed by David Hilbert at the Paris conference of the International Congress of Mathematicians in 1900. The problem is to determine the upper

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http://dx.doi.org/10.1016/j.jde.2016.04.025