Persistence of periodic traveling waves and Abelian integrals

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Abstract

It is well known that the existence of traveling wave solutions (TWS) for many partial differential equations (PDE) is a consequence of the fact that an associated planar ordinary differential equation (ODE) has certain types of solutions defined for all time. In this paper we address the problem of persistence of TWS of a given PDE under small perturbations. Our main results deal with the situation where the associated ODE has a center and, as a consequence, the original PDE has a continuum of periodic traveling wave solutions. We prove that the TWS that persist are controlled by the zeroes of some Abelian integrals. We apply our results to several famous PDE, like the Ostrovsky, Klein-Gordon, sine-Gordon, Korteweg-de Vries, Rosenau-Hyman, Camassa-Holm, and Boussinesq equations.
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