PACIFIC JOURNAL OF MATHEMATICS
Vol. 226, No. 2, 2006

UPPER BOUNDS FOR THE NUMBER OF LIMIT CYCLES THROUGH LINEAR DIFFERENTIAL EQUATIONS

ARMENGOL GASULL AND HECTOR GIACOMINI

Consider the differential equation $\dot{x} = y$, $\dot{y} = h_0(x) + h_1(x)y + h_2(x)y^2 + y^3$ in the plane. We prove that if a certain solution of an associated linear ordinary differential equation does not change sign, there is an upper bound for the number of limit cycles of the system. The main ingredient of the proof is the Bendixson–Dulac criterion for ℓ-connected sets. Some concrete examples are developed.

1. Main results

Although second order ordinary differential equations of the form $\ddot{x} = f(x, \dot{x})$ are some of the easiest autonomous planar differential equations, most problems concerning the study of the number of periodic solutions remain open. For instance, even if we consider the *Kukles system* $\dot{x} = y$, $\dot{y} = f_3(x, y)$, where f_3 is a polynomial of degree at most 3, the maximum number of limit cycles that it can have is still unknown.

This paper deals with the problem of finding methods to establish upper bounds for the number of limit cycles of planar differential equations of the form

$$\dot{x} = y, \quad \dot{y} = h_0(x) + h_1(x)y + h_2(x)y^2 + y^3, \tag{1–1}$$

where the functions h_i are smooth enough.

The proof of our main result is based on the use of the generalized Bendixson–Dulac criterion for ℓ-connected sets. Recall that an open subset U of $\mathbb{R}^2$ is said to be *ℓ-connected* if its fundamental group $\pi_1(U)$ is the free group in ℓ generators. This method has already been used with similar goals by several authors; see for instance [Cherkas 1997; Lloyd 1979; Yamato 1979; Cherkas and Grin' 1997; 1998; Gasull and Giacomini 2002]. The novelty of our approach is that we are able to reduce the computation of an upper bound for the number of limit cycles of the

MSC2000: 34C07, 34C05, 34A30, 37C27.
Keywords: ordinary differential equation, limit cycle, Bendixson–Dulac criterion, linear ordinary differential equation.
The first author is partially supported by the DGES grant number BFM2002-04236 and CONACIT grant number 2001SGR-00173.