## Some Applications of the Extended Bendixson-Dulac Theorem

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**Abstract** During the last years the authors have studied the number of limit cycles of several families of planar vector fields. The common tool has been the use of an extended version of the celebrated Bendixson-Dulac Theorem. The aim of this work is to present an unified approach of some of these results, together with their corresponding proofs. We also provide several applications.

## 1 The Bendixson-Dulac Theorem

Ivar Bendixson and Henri Dulac are the fathers of the today known as Bendixson-Dulac Theorem. The classical version of this theorem appears in most textbooks on differential equations; see [14, 31, 37, 38] with many applications. Let us recall it. Consider a  $\mathscr{C}^1$ -planar differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

defined in some open simply connected subset  $\mathscr{U} \subset \mathbb{R}^2$ , and set X = (P, Q). Assume that there exists a  $\mathscr{C}^1$  function  $D : \mathscr{U} \to \mathbb{R}$ , such that

$$\operatorname{div}(DX)|_{\mathscr{U}} = \left. \frac{\partial(D(x, y)P(x, y))}{\partial x} + \frac{\partial(D(x, y)Q(x, y))}{\partial y} \right|_{\mathscr{U}} \ge 0 \quad (\text{or} \le 0),$$

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