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Solving polynomials with ordinary differential equations

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Abstract

In this work we consider a given root of a family of n-degree polynomials as a one-variable function that depends only on the independent term. Then we prove that this function satisfies several ordinary differential equations (ODE). More concretely, it satisfies several simple separated variables ODE, a first order generalized Abel ODE of degree $n-1$ and an $(n-1)$-th order linear ODE. Although some of our results are not new, our approach is simple and self-contained. For $n = 2, 3$ and 4 we recover, from these ODE, the classical formulas for solving these polynomials.

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1. Introduction and main results

It is known that although general polynomial equations of degree $n \geq 5$ cannot be solved by radicals, their roots can be obtained in terms of elliptic or hyperelliptic functions, their inverses or other transcendental functions, like hypergeometric or theta

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