EFFECTIVE CONSTRUCTION OF POINCARÉ–BENDIXSON REGIONS

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Abstract. This paper deals with the problem of location and existence of limit cycles for real planar polynomial differential systems. We provide a method to construct Poincaré–Bendixson regions by using transversal curves, that enables us to prove the existence of a limit cycle that has been numerically detected. We apply our results to several known systems, like the Brusselator one or some Liénard systems, to prove the existence of the limit cycles and to locate them very precisely in the phase space. Our method, combined with some other classical tools can be applied to obtain sharp bounds for the bifurcation values of a saddle-node bifurcation of limit cycles, as we do for the Rychkov system.

1. Introduction

We consider real planar polynomial differential systems of the form

$$\dot{x} = dx/dt = P(x,y), \quad \dot{y} = dy/dt = Q(x,y), \tag{1}$$

where $P(x,y)$ and $Q(x,y)$ are real polynomials. We denote by $X = (P,Q)$ the vector field associated to (1) and $z = (x,y)$. So, (1) can be written as $\dot{z} = X(z)$.

When dealing with system (1) one of the main problems is to determine the number and location of its limit cycles. Recall that a limit cycle is an isolated periodic orbit of the system. For a given vector field, when it is not very near of a bifurcation, the limit cycles can usually be detected by numerical methods. A bifurcation is a qualitative change in the behaviour of a vector field as a parameter of the system is varied. This phenomenon can involve a change in the stability of a limit cycle or the creation or destruction of one or more limit cycles. If a periodic orbit is stable (unstable), then forward (backward) numerical integration of a trajectory with an initial condition in its basin of attraction will converge to the periodic orbit as $t \to \infty$ ($t \to -\infty$).

Once for a given vector field a limit cycle is numerically detected there are several methods to prove its existence. Some of them are based on Fixed

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