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A proof of Perko's conjectures for the Bogdanov–Takens system ☆

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ABSTRACT

The Bogdanov–Takens system has at most one limit cycle and, in the parameter space, it exists between a Hopf and a saddle-loop bifurcation curves. The aim of this paper is to prove the Perko's conjectures about some analytic properties of the saddle-loop bifurcation curve. Moreover, we provide sharp piecewise algebraic upper and lower bounds for this curve.

1. Introduction

The Bogdanov–Takens system

$$\begin{cases} x' = y, \\ y' = -n + by + x^2 + xy, \end{cases}$$

has been introduced in [1,17,18]. It provides a universal unfolding of a cusp point of codimension 2 and it is considered in many basic text books on bifurcation theory; see for instance [3,8,12]. Some

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