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Some results on homoclinic and heteroclinic connections in planar systems

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Abstract

Consider a family of planar systems depending on two parameters (n, b) and having at most one limit cycle. Assume that the limit cycle disappears at some homoclinic (or heteroclinic) connection when $\Phi(n, b) = 0$. We present a method that allows us to obtain a sequence of explicit algebraic lower and upper bounds for the bifurcation set $\Phi(n, b) = 0$. The method is applied to two quadratic families, one of them is the well-known Bogdanov–Takens system. One of the results that we obtain for this system is the bifurcation curve for small values of n, given by $b = \frac{5}{7}n^{1/2} + \frac{72}{2401}n - \frac{30024}{45294865}n^{3/2} - \frac{2352961656}{111083339166925}n^2 + O(n^{5/2})$. We obtain the new three terms from purely algebraic calculations, without evaluating Melnikov functions.

Mathematics Subject Classification: 34C23, 34C25, 34C37, 37C27

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Consider a smooth family of planar differential equations $(\dot{x}, \dot{y}) = (P(x, y; n, b), Q(x, y; n, b))$, $(n, b) \in \mathbb{R}^2$, for which the existence of at most one limit cycle is already known and moreover all the bifurcations occurring in the family are well understood. For this family we could say that the *Qualitative theory of ordinary differential equations* has achieved

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