DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS
Volume **33**, Number **8**, August **2013**

doi:10.3934/dcds.2013.33.3567

pp. **3567–3582**

EXPLICIT UPPER AND LOWER BOUNDS FOR THE TRAVELING WAVE SOLUTIONS OF FISHER-KOLMOGOROV TYPE EQUATIONS

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(Communicated by Adrian Constantin)

Abstract. It is well-known that the existence of traveling wave solutions for reaction-diffusion partial differential equations can be proved by showing the existence of certain heteroclinic orbits for related autonomous planar differential equations. We introduce a method for finding explicit upper and lower bounds of these heteroclinic orbits. In particular, for the classical Fisher-Kolmogorov equation we give rational upper and lower bounds which allow to locate these solutions analytically and with very high accuracy. These results allow one to construct analytical approximate expressions for the traveling wave solutions with a rigorous control of the errors for arbitrary values of the independent variables. These explicit expressions are very simple and tractable for practical purposes. They are constructed with exponential and rational functions.

1. **Introduction and main results.** Reaction-diffusion equations have been used to model a great variety of phenomena in fluid dynamics, dendritic growth, population growth, chemical reactions and biological models. Some solutions are particularly important to describe the dynamics of such systems, the so-called traveling wave solutions or front solutions. The traditional approach to computing the traveling waves consists of finding the explicit solutions for the front. There has been an extensive interest in looking for particular solutions of reaction-diffusion equations using different methods such as symmetry reductions, direct methods, the Painlevé test and the inverse scattering transform, see [5, 6, 9, 13].

2010 *Mathematics Subject Classification.* 34C37, 35C07, 37C29.

Key words and phrases. Traveling wave, heteroclinic orbit, Fisher-Kolmogorov equation, reaction-diffusion partial differential equation, invariant manifold.

The first and third authors are partially supported by a MCYT/FEDER grant number MTM2008-03437 and by a CIRIT grant number 2009SGR 410.