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CENTER PROBLEM FOR SYSTEMS WITH TWO MONOMIAL NONLINEARITIES

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ABSTRACT. We study the center problem for planar systems with a linear center at the origin that in complex coordinates have a nonlinearity formed by the sum of two monomials. Our first result lists several centers inside this family. To the best of our knowledge this list includes a new class of Darboux centers that are also persistent centers. The rest of the paper is dedicated to try to prove that the given list is exhaustive. We get several partial results that seem to indicate that this is the case. In particular, we solve the question for several general families with arbitrary high degree and for all cases of degree less or equal than 19. As a byproduct of our study we also obtain the highest known order for weak-foci of planar polynomial systems of some given degrees.

1. Introduction and statement of the main results. The center-focus problem consists in distinguishing whether a monodromic singular point is a center or a focus. For singular points with imaginary eigenvalues, usually called *nondegenerate singular points*, this problem was already solved by Poincaré and Lyapunov, see [19, 21, 22]. The solution consists in computing several quantities called commonly the *Poincaré–Lyapunov constants*, and study whether they are zero or not. There are different methods to compute them, for a brief survey of these methods see [4, 10, 12, 13, 20] and the references therein.

Despite the existence of many methods, the solution of the center-focus problem for simple families, like for instance the complete cubic systems or the quartic systems with homogeneous nonlinearities, has resisted all the attempts, see for instance [11, 18, 24]. For this reason, in this paper and following [17], we propose to push on this question in another direction. We study this problem for a natural

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