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LINEARIZATION OF PLANAR HOMEOMORPHISMS WITH A COMPACT ATTRACTOR

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ABSTRACT. Kerékjártó proved in 1934 that a planar homeomorphism with an asymptotically stable fixed point is conjugated, on its basin of attraction, to one of the maps $z \mapsto z/2$ or $z \mapsto \overline{z}/2$, depending on whether f preserves or reverses the orientation. We extend this result to planar homeomorphisms with a compact attractor.

1. Introduction

Consider the discrete dynamical system generated by a planar homeomorphism f. It is well-known that if f has an asymptotically stable fixed point, then its basin of attraction \mathcal{U} is an open and simply connected subset of the plane. Moreover, Kerékjártó ([7], [8]) proved that f restricted to \mathcal{U} is either conjugated to $L_1(z) = z/2$ or to $L_2(z) = \overline{z}/2$ in \mathbb{C} , depending on whether f preserves or reverses the orientation. A different proof of this result is also given in [4]. This result has been extended, with clear modifications, to \mathbb{R}^3 in [5] and to \mathbb{R}^m for $m \neq 4, 5$ in [6], when f preserves orientation.

In this paper we will focus on the planar case and we extend Kerékjártó's result to the case where f has a compact attractor. To state our result we need

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