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LIMIT CYCLES FOR GENERALIZED ABEL EQUATIONS*

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This paper deals with the problem of finding upper bounds on the number of periodic solutions of a class of one-dimensional nonautonomous differential equations: those with the right-hand sides being polynomials of degree n and whose coefficients are real smooth one-periodic functions. The case $n = 3$ gives the so-called *Abel equations* which have been thoroughly studied and are well understood. We consider two natural generalizations of Abel equations. Our results extend previous works of Lins Neto and Panov and try to step forward in the understanding of the case $n > 3$. They can be applied, as well, to control the number of limit cycles of some planar ordinary differential equations.

Keywords: Abel equation; limit cycles; planar differential equations.

1. Introduction and Main Results

Nonautonomous differential equations of type

$$\frac{dx}{dt} = S(t, x), \quad x \in \Omega \subset \mathbb{R}^n,\ t \in I \subset \mathbb{R}, \tag{1}$$

with additional boundary conditions are encountered in different problems like variational equations of periodic orbits of vector fields, plane autonomous ODE systems (see Sec. 4), control theory (see for instance [Fossas-Colet & Olm-Miras, 2002]), ... One is often interested in particular solutions $x(t)$ of (1) which are defined in a whole interval I (we take $I = [0, 1]$ throughout the paper) and such that $x(0) = x(1)$. In the case when S is one-periodic in t, observe that these solutions, which are closed when we consider (1) on the cylinder $\mathbb{R}^n \times [0, 1]$, can be called *periodic*. A periodic solution which is isolated in the set of all the periodic solutions of (1) is called a *limit cycle* of the differential equation.

One of the most challenging questions for Eq. (1) is the control of the number of limit cycles in families of equations. Is this number finite? Is it bounded?

Despite this interest, the simplest situations are not completely understood yet, as in the one-dimensional "polynomial" case,

$$\frac{dx}{dt} = a_n(t)x^n + a_{n-1}(t)x^{n-1} + \cdots + a_1(t)x + a_0(t), \tag{2}$$

where $x \in \mathbb{R}$, $t \in [0, 1]$ and $a_0, a_1, \ldots, a_n : \mathbb{R} \to \mathbb{R}$, are smooth one-periodic functions. The general

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