



Phase portrait of Hamiltonian systems with homogeneous nonlinearities¹

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1. Introduction

In this paper we study the phase portraits of Hamiltonian systems of the form

$$\dot{x} = -H_y(x, y),$$

$$\dot{y} = H_x(x, y), \tag{1}$$

where $H(x, y) = (x^2 + y^2)/2 + H_{n+1}(x, y)$, and H_{n+1} is a homogeneous polynomial of degree $n+1$. The solutions of system (1) are contained in the level curves $\{H(x, y)=h, h \in \mathbb{R}\}$. Furthermore, the origin is a nondegenerate center.

The knowledge of phase portraits of Hamiltonian systems is useful for several reasons. We comment on some of them in the sequel.

(i) Small polynomial perturbations of them give rise to polynomial systems with limit cycles. For instance, the cubic system with 11 limit cycles presented in [15]

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