QUALITATIVE THEORY OF DYNAMICAL SYSTEMS **4**, 329–352 (2004)
ARTICLE NO. 66

The Period Function for Second-Order Quadratic ODEs is Monotone*

Armengol Gasull

Dept. de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona. 08193 Bellaterra, Barcelona, Spain.
E-mail: gasull@mat.uab.es

Antoni Guillamon

Dept. de Matemàtica Aplicada I, Universitat Politècnica de Catalunya. Dr. Marañón, 44-50. 08028, Barcelona.
E-mail: antoni.guillamon@upc.es

Jordi Villadelprat

Dept. d'Enginyeria Informàtica i Matemàtiques, Universitat Rovira i Virgili, Av. dels Països Catalans 26, 43007 Tarragona, Spain.
E-mail: Jordi.Villadelprat@etse.urv.es

Submitted: January 17, 2003 *Accepted*: August 18, 2003

Dedicated to Professor Jorge Sotomayor on the occasion of his 60th birthday.

Very little is known about the period function for large families of centers. In one of the pioneering works on this problem, Chicone [3] conjectured that all the centers encountered in the family of second-order differential equations $\ddot{x} = V(x, \dot{x})$, being V a quadratic polynomial, should have a monotone period function. Chicone solved some of the cases but some others remain still unsolved. In this paper we fill up these gaps by using a new technique based on the existence of Lie symmetries and presented in [8]. This technique can be used as well to reprove all the cases that were already solved, providing in this way a compact proof for all the quadratic second-order differential equations. We also prove that this property on the period function is no longer true when V is a polynomial which nonlinear part is homogeneous of degree $n > 2$.

Key Words: Period function, Second-order quadratic ODEs

* Partially supported by the DGES grant number BFM2002-04236 and CONACIT grant number 2001SGR-00173.