J. Math. Anal. Appl. 422 (2015) 376–396

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

On the norming constants for normal maxima

Armengol Gasull, Maria Jolis *, Frederic Utzet

Departament de Matemàtiques, Edifici C, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

ARTICLE INFO

Article history:
Received 28 April 2014
Available online 19 August 2014
Submitted by V. Pozdnyakov

Keywords:
Extreme value theory
Gaussian law
Lambert W function

ABSTRACT

Given n independent standard normal random variables, it is well known that their maxima M_n can be normalized such that their distribution converges to the Gumbel law. In a remarkable study, Hall proved that the Kolmogorov distance d_n between the normalized M_n and its associated limit distribution is less than $3/\log n$. In the present study, we propose a different set of norming constants that allow this upper bound to be decreased with $d_n \leq C(m)/\log n$ for $n \geq m \geq 5$. Furthermore, the function $C(m)$ is computed explicitly, which satisfies $C(m) \leq 1$ and $\lim_{m\to\infty} C(m) = 1/3$. As a consequence, some new and effective norming constants are provided using the asymptotic expansion of a Lambert W type function.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

Let $X_1, \ldots, X_n$ be i.i.d. standard normal random variables and denote by $M_n = \max\{X_1, \ldots, X_n\}$ their maximum. The normal law is in the domain of attraction for the maxima of the Gumbel law, i.e., there are sequences of norming constants, $a_n > 0$ and $b_n \in \mathbb{R}$, such that for every $x \in \mathbb{R}$,

$$\lim_n \Phi^n(a_n x + b_n) = \Lambda(x), \tag{1}$$

where Φ is the distribution function of a standard normal law and

$$\Lambda(x) = \exp\{-e^{-x}\}, \quad x \in \mathbb{R},$$

is the distribution function of a Gumbel random variable G.

* Corresponding author.
E-mail addresses: gasull@mat.uab.cat (A. Gasull), mjolis@mat.uab.cat (M. Jolis), utzet@mat.uab.cat (F. Utzet).

http://dx.doi.org/10.1016/j.jmaa.2014.08.025
0022-247X/© 2014 Elsevier Inc. All rights reserved.