

LIMIT CYCLES IN THE HOLLING-TANNER MODEL

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Abstract

This paper deals with the following question: does the asymptotic stability of the positive equilibrium of the Holling-Tanner model imply it is also globally stable? We will show that the answer to this question is negative. The main tool we use is the computation of Poincaré-Lyapunov constants in case a weak focus occurs. In this way we are able to construct an example with two limit cycles.

1. Introduction

The two main types of interaction between any pair of biological species, which are of interest to the ecologist, are either when they are competing together for some common source of food supply, or when one of the species preys upon the other. In this paper we will restrict our attention to the latter case.

The existence and the number of isolated periodic solutions (limit cycles) is one of the most delicate problems connected with two-dimensional predator-prey models.

One of the first examples of a biological system modelling the interaction between prey and predators was formulated by Lotka in 1925 [11] and Volterra in 1927 [17]:

$$(1.1) \quad \begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= -\delta y + \gamma xy. \end{aligned}$$

In system (1.1) $x(t)$ and $y(t)$ denote prey and predator densities respectively, as functions of time. Furthermore, all constants are assumed to be positive. Obviously, the attention is restricted to $x \geq 0$, $y \geq 0$.