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Upper bounds for the number of zeroes for some Abelian integrals

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ABSTRACT

Consider the vector field $x' = -yG(x,y), y' = xG(x,y)$, where the set of critical points $\{G(x,y) = 0\}$ is formed by K straight lines, not passing through the origin and parallel to one or two orthogonal directions. We perturb it with a general polynomial perturbation of degree n and study the maximum number of limit cycles that can bifurcate from the period annulus of the origin in terms of K and n. Our approach is based on the explicit computation of the Abelian integral that controls the bifurcation and on a new result for bounding the number of zeroes of a certain family of real functions. When we apply our results for $K \leq 4$ we recover or improve some results obtained in several previous works.

1. Introduction

The problem of determining the number of limit cycles bifurcating from the period annulus of a system

$$\begin{cases} \dot{x} = -yG(x,y) + \varepsilon P(x,y), \\ \dot{y} = -xG(x,y) + \varepsilon Q(x,y), \end{cases} \tag{1}$$

where $P(x,y)$, $Q(x,y)$ are polynomials of a given degree, $G(x,y)$ satisfies $G(0,0) \neq 0$ and ε is a small parameter, has been widely studied (see for instance [1–8]). Among this type of systems we will be concerned with those having

$$G(x,y) = \prod_{j=1}^{K_1}(x - \mathrm{a}_j)\prod_{\ell=1}^{K_2}(y - \mathrm{b}_\ell), \tag{2}$$

where a_j and b_ℓ are real numbers with $\mathrm{a}_i \neq \mathrm{a}_j$ and $\mathrm{b}_i \neq \mathrm{b}_j$ for $i \neq j$. The unperturbed system ($\varepsilon = 0$) presents a centre at the origin and any line $x = \mathrm{a}_j$ or $y = \mathrm{b}_\ell$ constitutes an invariant set of singular points of the system. This invariant set is formed by parallel and/or orthogonal invariant lines.

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