



Rational Parameterizations Approach for Solving Equations in Some Dynamical Systems Problems

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Abstract

We show how the use of rational parameterizations facilitates the study of the number of solutions of many systems of equations involving polynomials and square roots of polynomials. We illustrate the effectiveness of this approach, applying it to several problems appearing in the study of some dynamical systems. Our examples include Abelian integrals, Melnikov functions and a couple of questions in Celestial Mechanics: the computation of some relative equilibria and the study of some central configurations.

Keywords Bifurcation · Resultant · Rational parameterization · Abelian integral · Poincaré–Melnikov–Pontryagin function · Relative equilibria · Central configuration

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1 Introduction

A very common way to solve equations involving polynomials and square roots of polynomials consists in squaring them several times until they become polynomials and look for their zeros using a numerical method. Besides the fact that the degree of such polynomials increases with any squaring, there exists a second inconvenience

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