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A new Chebyshev family with applications to Abel equations

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ABSTRACT

We prove that a family of functions defined through some definite integrals forms an extended complete Chebyshev system. The key point of our proof consists of reducing the study of certain Wronskians to the Gram determinants of a suitable set of new functions. Our result is then applied to give upper bounds for the number of isolated periodic solutions of some perturbed Abel equations.

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1. Introduction and main results

In this paper we introduce the family of analytic functions

$$I_{k,\alpha}(y) := \int_{a}^{b} \frac{g^{k}(t)}{(1 - yg(t))^{\alpha}} dt,$$
(1)

for k = 0, 1, ..., n, and prove that it is an *extended complete Chebyshev system* (for short, an ECT-system). In contrast to what is commonly done in other papers, no explicit integration of functions $I_{k,\alpha}$ is needed. In fact, our proof is based on the standard characterization of ECT-systems through

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