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Limit cycles appearing from the perturbation of a system with a multiple line of critical points

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ABSTRACT

Consider planar ordinary differential equations of the form $\dot{x} = -yC(x, y), \dot{y} = xC(x, y)$, where $C(x, y)$ is an algebraic curve. We are interested in knowing whether the existence of multiple factors for C is important or not when we study the maximum number of zeros of the Abelian integral M that controls the limit cycles that bifurcate from the period annulus of the origin when we perturb it with an arbitrary polynomial vector field. With this aim, we study in detail the case $C(x, y) = (1 - y)^m$, where m is a positive integer number and prove that m has essentially no impact on the number of zeros of M. This result improves the known studies on M. One of the key points of our approach is that we obtain a simple expression of M based on some successive reductions of the integrals appearing during the procedure.

1. Introduction and main result

Consider the family of planar systems

$$\begin{cases} \dot{x} = -yC(x, y) + \varepsilon P(x, y), \\ \dot{y} = xC(x, y) + \varepsilon Q(x, y), \end{cases} \tag{1}$$

where P, Q and C are real polynomials, $C(0, 0) \neq 0$, and ε is a small real parameter. It is well known that the number of zeros of the Abelian integral

$$M(r) = \int_{\gamma_r} \frac{Q(x, y)\,\mathrm{d}x - P(x, y)\,\mathrm{d}y}{C(x, y)}, \tag{2}$$

where $\gamma_r = \{(x, y) : x^2 + y^2 = r^2\}$, controls the number of limit cycles of (1) that bifurcate from the periodic orbits of the unperturbed system (1) with $\varepsilon = 0$; see [1]. Note that since the sets γ_r are circles, the Abelian integral $M(r)$ is simpler that in the general weak Hilbert's 16th Problem; see [2].

The problem of finding upper and lower bounds for the number of zeros of $M(r)$, when P and Q are arbitrary polynomials of a given degree, say n, and C is a particular polynomial, has been faced in several recent papers. The cases where the curve $\{C(x, y) = 0\}$ is one line, two parallel lines, two orthogonal lines, k lines, parallel to two orthogonal directions, or k isolated points are some of these situations; see [3–7], respectively. Note that none of these algebraic curves has a multiple factor.

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