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Limit cycles for 3-monomial differential equations

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ABSTRACT

We study planar polynomial differential equations that in complex coordinates write as $\dot z = Az + Bz^k\bar z^l + Cz^m\bar z^n$. We prove that for each $p \in \mathbb{N}$ there are differential equations of this type having at least p limit cycles. Moreover, for the particular case $\dot z = Az + B\bar z + Cz^m\bar z^n$, which has homogeneous nonlinearities, we show examples with several limit cycles and give a condition that ensures uniqueness and hyperbolicity of the limit cycle.

1. Introduction and main results

The celebrated second part of the Hilbert's 16th problem [17] consists in determining a uniform upper bound on the number of limit cycles of all polynomial differential systems of degree N, see for instance [19,21] and the references therein. This problem is still open even for the quadratic case, $N = 2$. Due to its extreme difficulty, usually people fix a subclass of polynomial differential equations, namely: quadratic, cubic, Kukles, Liénard, homogeneous nonlinearities,..., and then try to advance in the question restricted to the selected family. This paper goes in a similar direction, we consider a simple class of polynomial systems, but instead of fixing the degree, we fix a short number of monomials once the system is written in complex coordinates, and then we study its number of limit cycles.

To be more precise, consider two dimensional real differential systems,

$$\frac{dx}{dt} = \dot x = P(x,y), \qquad \frac{dy}{dt} = \dot y = Q(x,y), \quad (x,y) \in \mathbb{R}^2, \quad t \in \mathbb{R},$$

with P and Q polynomials. They can also be written as

$$\frac{dz}{dt} = \dot z = F(z,\bar z), \quad z \in \mathbb{C}, \quad t \in \mathbb{R},$$

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