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Further Considerations on the Number of Limit Cycles of Vector Fields of the Form $X(v) = Av + f(v)\, Bv$

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In Gasull, Llibre, and Sotomayor. (*J. Differential Equations*, in press) we studied the number of limit cycles of planar vector fields as in the title. The case where the origin is a node with different eigenvalues, which then resisted our analysis, is solved in this paper.

1. INTRODUCTION

In [3] we studied vector fields of the form

$$X(v) = Av + f(v)\, Bv, \tag{1}$$

where A and B are 2×2 matrices, $\det A \neq 0$ and $f\colon \mathbb{R}^2 \to \mathbb{R}$ is a smooth real function such that its expression in polar coordinates is $f(r\cos\theta, r\sin\theta) = r^D \bar{f}(\theta)$ with $D \geqslant 1$. Roughly, we shall say that f is a *homogeneous function of degree* $\mathscr{D}$. There is one hypothesis for the matrices A and B. This hypothesis states that $(JB)_s$ and $(B^t JA)_s$ are definite and have the same sign (for a 2×2 matrix C let C^t denote the transpose of C, $C_s = (C + C^t)/2$ and $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$). When the matrices A and B satisfy this property we shall say that system (1) satisfies *hypothesis* H_1.

We shall say that f is *indefinite* if f takes both positive and negative values.

For vector fields (1) we described in [3] their phase portrait, determin-

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