

Continua of Periodic Points for Planar Integrable Rational Maps

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Abstract

We present three alternative methodologies to find continua of periodic points with a prescribed period for rational maps having rational first integrals. The first two have been already used by other authors and apply when the maps are birational and the generic level sets of the corresponding first integrals have either genus 0 or 1. As far as we know, the third one is new and it works for rational maps without imposing topological properties to the invariant level sets. It is based on a computational point of view, and relies on the use of resultants in a suitable setting. We apply them to several examples, including the 2-periodic Lyness composition maps and some of the celebrated McMillan–Gumovski–Mira maps.

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1 Introduction

A planar *rational* map $F : \mathcal{U} \rightarrow \mathbb{K}^2$, where $\mathcal{U} \subseteq \mathbb{K}^2$ is an open set and $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, is called *birational* if it has a rational inverse F^{-1} . Such a map is *integrable* if there exists

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