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EFFECTIVE COMPUTATION OF THE FIRST LYAPUNOV QUANTITIES FOR A PLANAR DIFFERENTIAL EQUATION

Abstract. We take advantage of the complex structure to compute in a short way and without using any computer algebra system the Lyapunov quantities V_3 and V_5 for a general smooth planar system.

1. Introduction. Consider the differential equation $(\dot{x}, \dot{y}) = (f(x, y), g(x, y))$, $(x, y) \in \mathbb{R}^2$, in the plane where f and g are analytic functions satisfying $f(0, 0) = g(0, 0) = 0$. It is well known that when the origin is a non-hyperbolic critical point of focus type the study of its stability can be reduced to the computation of the so called Lyapunov quantities, $V_{2k+1}, k = 1, 2, \ldots$; see [ALGM] for more details. By making a linear change of coordinates and a rescaling of the time variable if necessary, the planar differential equation can be written as

$$\dot{z} = F(z, \overline{z}) = iz + \sum_{k=2}^{\infty} F_k(z, \overline{z}), \tag{1}$$

where $z = x + iy = \mathrm{Re}(z) + i\,\mathrm{Im}(z)$, and F_k is a complex homogeneous polynomial of degree k.

In this paper we make some modifications in the standard techniques explained in [ALGM] to obtain the Lyapunov quantities. These modifications simplify their effective computation. The main idea is to keep the complex structure of (1) during all the process.

In Section 2 we give some preliminary results and in Section 3 we prove:

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