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Simple Examples of One-Parameter Planar Bifurcations

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1. INTRODUCTION AND STATEMENT OF MAIN RESULTS

In this paper we give simple and low degree examples of one-parameter polynomial families of planar differential equations which present generic, codimension one, isolated, compact bifurcations. In contrast with some examples which appear in the usual text books each bifurcation occurs when the bifurcation parameter is zero. We study the total number of limit cycles that the examples present and we also make their phase portraits on the Poincaré sphere.

Consider a differential equation $\dot{x} = X(x)$, where X belongs to a family of smooth vector fields $\mathcal{F}$ and $x \in \mathbb{R}^2$. Fix a compact set $K \subset \mathbb{R}^2$ and a distance d between the elements of $\mathcal{F}$ restricted to K. Following [1], we will say that X gives a dynamical system of first degree of structural instability in K if it is structurally unstable in K, whereas any sufficiently d-close system inside $\mathcal{F}$ is either structurally stable in K or topologically equivalent to X. It is well known, (see also [1]), that for generic families of one-parameter differential equations passing through a differential equation of first degree of structural instability only the following compact bifurcations appear:

(1) Saddle-node.

(2) Saddle connection.

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