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Fixed and moving limit cycles for Liénard equations

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Abstract
We consider a family of planar vector fields that writes as a Liénard system in suitable coordinates. It has a fixed closed invariant curve that often contains periodic orbits of the system. We prove a general result that gives the hyperbolicity of these periodic orbits, and we also study the coexistence of them with other periodic orbits. Our family contains the celebrated Wilson polynomial Liénard equation, as well as all polynomial Liénard systems having hyperelliptic limit cycles. As an illustrative example, we study in more detail a natural 1-parametric extension of Wilson example. It has at least two limit cycles, one of them fixed and algebraic and the other one moving with the parameter, presents a transcritical bifurcation of limit cycles and for a given parameter has a non-hyperbolic double algebraic limit cycle. In order to prove that for some values of the parameter the system has exactly two hyperbolic limit cycles, we use several suitable Dulac functions.

Keywords Liénard equation · Limit cycle · Bifurcations · Invariant algebraic curve

Mathematics Subject Classification Primary 34C07; Secondary 37C23 · 34C25 · 37C27

1 Introduction and main results

Liénard equations

$$\ddot{x} + f(x)\dot{x} + g(x) = 0, \tag{1}$$

are widely studied as models of physical, chemical, biological or electrical phenomena. A particular interesting case is when they contain algebraic limit cycles or polycycles. Wilson's paper [27] is maybe one of the first works dealing with this question. In the series of papers [2, 19,27,31], the study of hyperelliptic solutions $(y + R(x))^2 + S(x) = 0$ of polynomial Liénard

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