Euler-Jacobi formula for double points and applications to quadratic and cubic systems*

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Abstract

We prove a generalization of the Euler-Jacobi formula for double points. We apply it to study the distribution of the critical points for quadratic and cubic systems, when one of these points is double.

1 Introduction.

Consider a planar polynomial vector field $X = (P, Q)$, with $\deg P = n$, $\deg Q = m$. Assume that X has exactly nm simple critical points (real or complex). In this situation the Euler-Jacobi formula gives an algebraic relation between the critical points of X and their indices, see Section 3. In this paper we prove a generalization of the formula, allowing X to have double points. In fact in Section 2 we give necessary and sufficient conditions for a critical point be double and in Section 3 we give a proof of this new Euler-Jacobi formula. The method used could be applied to get a general formula for more degenerate critical points, but the computations involved would increase very much. We have not made these computations here.

Finally, Section 4 deals with applications of the formula obtained to study the distribution of the critical points of quadratic vector fields ($n = m = 2$) and a sub class of cubic vector fields ($n = 2, m = 3$). The result obtained can be considered as a continuation of the paper [3]. For instance Theorem 4.1 is a generalization of the well-known Berlinskii's Theorem, see [2, 5], and can be interpreted as the limit case of that result.

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