Small-Amplitude Limit Cycles in Liénard Systems via Multiplicity[1]

Armengol Gasull and Joan Torregrosa

Dept. de Matemàtiques, Universitat Autònoma de Barcelona, Edifici Cc 08193 Bellaterra, Barcelona, Spain
E-mail: gasull@mat.uab.es, torre@mat.uab.es

A recent paper of Christopher and Lloyd reduces the computation of the order of degeneracy of a weak focus for a polynomial Liénard system $\dot{x} = y - F(x)$, $\dot{y} = -g(x)$ to the computation of the multiplicity of a polynomial map. In this paper, we first take advantage of that approach to obtain new lower and upper bounds for the maximum order of degeneracy of the origin in terms of the degrees of F' and g. Later on, we implement an algorithm to compute this maximum order for concrete values of these degrees. As far as we know we enlarge the set of degrees for which this maximum order was known. Finally we extend Christopher and Lloyd's result to analytic degenerate (or not) Liénard equations. © 1999 Academic Press
   Key Words: Liénard equation; limit cycles; multiplicity.

1. INTRODUCTION

One of the second order differential equations that has attracted the interest of the mathematicians during these last decades is the equation of Liénard

$$\ddot{x} + f(x)\,\dot{x} + g(x) = 0, \tag{1}$$

or its equivalent first order system

$$\dot{x} = y - F(x), \qquad \dot{y} = -g(x), \tag{2}$$

where $y = \dot{x} + F(x)$, and $F(x) = \int_0^x f(s)\,ds$. Apart from the fact that (1) frequently appears in applications, it is studied because many other systems can be transformed into this form; see for instance [11, 21].

186