Computational and Applied Mathematics Vol. 20, N. 1-2, pp. 1–29, 2001 Copyright © 2001 SBMAC

A new approach to the computation of the Lyapunov Constants*

ARMENGOL GASULL and JOAN TORREGROSA

Dept. de Matemàtiques, Universitat Autònoma de Barcelona Edifici Cc 08193 Bellaterra, Barcelona, Spain E-mail: gasull@mat.uab.es / torre@mat.uab.es

Abstract. The problem of distinguishing whether a critical point of an analytic planar vector field with pure imaginary eigenvalues is a center or a focus was already solved by Lyapunov by searching an appropriate Lyapunov function. During the last years different methods have been developed to improve his approach. In this paper we give a new one. It is based on the study of the higher order Melnikov functions of a perturbation of the Hamiltonian system associated to $H(x, y) = (x^2 + y^2)/2$. From our point of view, our method has the following advantages: it is easy to be implemented; it allows to express the Lyapunov constants as words; it permits to prove – in an easy way – the algebraic properties of the Lyapunov constants when they are regarded as polynomials in the coefficients of the systems; it can also be applied to special types of degenerate monodromic critical points and, finally, it allows to relate the cyclicity of the critical point with the number of limit cycles which appear from the level curves of the linear center by using small perturbations.

Mathematical subject classification: 34C25, 34C07.

Key words: Center-focus problem, Lyapunov constants, Melnikov functions.

1 Introduction and main results

The center-focus problem (that is to distinguish whether a critical point of an analytic planar vector field with pure imaginary eigenvalues is a center or a focus) was already solved by Lyapunov. He gave a method for obtaining a set of

^{#07/01.} Received: ??/I/01.

^{*}Dedicated to Professor Mauricio Peixoto in occasion of his 80th birthday.

Partially supported by the DGICYT grant number PB96-1153 and CONACIT grant number 1999SGR 00349.