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CENTER-FOCUS PROBLEM FOR DISCONTINUOUS PLANAR DIFFERENTIAL EQUATIONS

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We study the center-focus problem as well as the number of limit cycles which bifurcate from a weak focus for several families of planar discontinuous ordinary differential equations. Our computations of the return map near the critical point are performed with a new method based on a suitable decomposition of certain one-forms associated with the expression of the system in polar coordinates. This decomposition simplifies all the expressions involved in the procedure. Finally, we apply our results to study a mathematical model of a mechanical problem, the movement of a ball between two elastic walls.

Keywords: Discontinuous ordinary differential equation; center problem; limit cycle; nonsmooth system; Lyapunov constant.

1. Introduction

There are many problems in science, and particularly in mechanics and engineering, where their mathematical modelization is given by a dynamical system whose right-hand side is not continuous or not differentiable, see for instance the classical book [Andronov *et al.*, 1987] or the new one [Kunze, 2000] and the references therein.

In this paper we study the following class of discontinuous planar systems of ordinary differential equations

$$(\dot{x},\dot{y})=\begin{cases}(-y+P^+(x,y),x+Q^+(x,y)) & \text{if } y\geq 0\,,\\ (-y+P^-(x,y),x+Q^-(x,y)) & \text{if } y\leq 0\,,\end{cases} \tag{1}$$

where P^+, Q^+, P^-, Q^- are analytic functions starting at least with second order terms.

The above system has the origin as a monodromic critical point. We are interested in the following two problems:

- The center-focus problem, i.e. to determine if the origin of system (1) is either a center, an attractor or a repeller.
- The cyclicity problem, that is, fix a class of systems of type (1) and determine the maximum number of limit cycles which bifurcate from the origin under the variation of the parameters inside this class of systems.

Our main contribution is the development of a new method to compute the Lyapunov constants — defined in the next section — for systems of type (1). This method gives a tool to solve the above problems. It is based on a suitable decomposition of certain one-forms associated with the expression of (1) in polar coordinates. Our decomposition is done in such a way that it simplifies the computations needed in all procedures. Furthermore, it is easy to be implemented in a computer algebraic