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# The number of polynomial solutions of polynomial Riccati equations

Armengol Gasull [a], Joan Torregrosa [a,*], Xiang Zhang [b]

[a] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*
[b] *School of Mathematical Sciences and MOE-LSC, Shanghai Jiao Tong University, Shanghai 200240, PR China*

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**Abstract**

Consider real or complex polynomial Riccati differential equations $a(x)\dot{y} = b_0(x) + b_1(x)y + b_2(x)y^2$ with all the involved functions being polynomials of degree at most $\eta$. We prove that the maximum number of polynomial solutions is $\eta + 1$ (resp. 2) when $\eta \geq 1$ (resp. $\eta = 0$) and that these bounds are sharp.

For real trigonometric polynomial Riccati differential equations with all the functions being trigonometric polynomials of degree at most $\eta \geq 1$ we prove a similar result. In this case, the maximum number of trigonometric polynomial solutions is $2\eta$ (resp. 3) when $\eta \geq 2$ (resp. $\eta = 1$) and, again, these bounds are sharp.

Although the proof of both results has the same starting point, the classical result that asserts that the cross ratio of four different solutions of a Riccati differential equation is constant, the trigonometric case is much more involved. The main reason is that the ring of trigonometric polynomials is not a unique factorization domain.


* Corresponding author.
*E-mail addresses:* gasull@mat.uab.cat (A. Gasull), torre@mat.uab.cat (J. Torregrosa), xzhang@sjtu.edu.cn (X. Zhang).

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