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Vector fields with homogeneous nonlinearities and many limit cycles

Armengol Gasull [a], Jiang Yu [b], Xiang Zhang [c,*]

[a] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*
[b] *Department of Mathematics, Shanghai Jiao Tong University, Shanghai, 200240, PR China*
[c] *Department of Mathematics, and MOE-LSC, Shanghai Jiao Tong University, Shanghai, 200240, PR China*

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Abstract

Consider planar real polynomial differential equations of the form $\dot{\mathbf{x}} = L\mathbf{x} + X_n(\mathbf{x})$, where $\mathbf{x} = (x, y) \in \mathbb{R}^2$, L is a 2×2 matrix and X_n is a homogeneous vector field of degree $n > 1$. Most known results about these equations, valid for infinitely many n, deal with the case where the origin is a focus or a node and give either non-existence of limit cycles or upper bounds of one or two limit cycles surrounding the origin. In this paper we improve some of these results and moreover we show that for $n \geq 3$ odd there are equations of this form having at least $(n + 1)/2$ limit cycles surrounding the origin. Our results include cases where the origin is a focus, a node, a saddle or a nilpotent singularity. We also discuss a mechanism for the bifurcation of limit cycles from infinity.

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* Corresponding author.
E-mail addresses: gasull@mat.uab.cat (A. Gasull), jiangyu@sjtu.edu.cn (J. Yu), xzhang@sjtu.edu.cn (X. Zhang).

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