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ON A FAMILY OF POLYNOMIAL DIFFERENTIAL EQUATIONS HAVING AT MOST THREE LIMIT CYCLES

ARMENGOL GASULL AND YULIN ZHAO

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Abstract. We prove the existence of at most three limit cycles for a family of planar polynomial differential equations. Moreover we show that this upper bound is sharp. The key point in our approach is that the differential equations of this family can be transformed into Abel differential equations.

1. Introduction and main results

Hilbert's 16th Problem has been one of the main problems in the qualitative theory of ordinary differential equations in the last century, and continues to attract widespread interest. It is concerned with the number and possible configurations of limit cycles for planar polynomial differential systems. This problem has not been solved even for the quadratic case.

In this paper we are interested in the study of the number of limit cycles of polynomial differential systems of the form

$$(1)\quad \begin{cases} \dot{x} &= x(P_{n-1}(x,y) + P_{n+2m-1}(x,y) + P_{n+3m-1}(x,y)) + Q_{n+m}(x,y), \\ \dot{y} &= y(P_{n-1}(x,y) + P_{n+2m-1}(x,y) + P_{n+3m-1}(x,y)) + R_{n+m}(x,y), \end{cases}$$

where the dot denotes the derivative with respect to the time t, n and m are positive natural numbers and $P_k(x,y), Q_k(x,y)$ and $R_k(x,y)$ are homogeneous polynomials of degree k. We introduce the new homogeneous polynomial

$$G_{n+m+1}(x,y) = xR_{n+m}(x,y) - yQ_{n+m}(x,y),$$

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