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SOLITARY TRAVELING WATER WAVES OF MODERATE AMPLITUDE

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We prove the existence of solitary traveling wave solutions for an equation describing the evolution of the free surface for waves of moderate amplitude in the shallow water regime. This nonlinear third-order partial differential equation arises as an approximation of the Euler equations, modeling the unidirectional propagation of surface water waves. We give a description of the solitary wave profiles by performing a phase plane analysis and study some qualitative features of the solutions.

Keywords: Solitary waves; moderate amplitude; homoclinic orbit.

Mathematics Subject Classification 2000: 76B25, 37C29, 34C99

1. Introduction

Ever since Scott Russell's first recorded observation of "the great wave of translation" [33], there has been growing interest in the study of solitary wave solutions of the equations for water waves. The existence theory for irrotational waves of small amplitude dates back to works of Krasovski, Lavrentiev and Ter-Krikorov [26, 27, 34], and was later improved by Friedrichs and Hyers [17], Beale [6] and Amick and Toland [4]. Although at the time no existence results for waves with arbitrary amplitude were available, Keady and Pritchard [24] proved that symmetric and monotone solitary wave solutions are necessarily waves of elevation which propagate at supercritical Froude number. It was shown by Amick and Toland [3] that such waves of elevation actually exist for all amplitudes from zero up to the solitary wave of greatest height and that they decay exponentially at infinity, under the assumption that the wave profile is symmetric and monotone from crest to trough. Craig and Sternberg [16] proved that any supercritical solitary wave solution is symmetric and decays monotonically to a constant on either side of the crest. More recently, results on existence, symmetry and regularity were obtained in the rotational case, cf. [20, 29], and the flow beneath an irrotational solitary wave was investigated in [11, 12]. In parallel with the aforementioned research on the exact water wave problem, the past fifty years have seen