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# A NOTE ON UNIQUENESS AND COMPACT SUPPORT OF SOLUTIONS IN A RECENT MODEL FOR TSUNAMI BACKGROUND FLOWS

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Abstract. We present an elementary proof of uniqueness for solutions of an initial value problem which is not Lipschitz continuous, generalizing a technique employed in [20]. This approach can be applied for a wide class of vorticity functions in the context of [6], where, departing from a recent model for the evolution of tsunami waves developed in [10], the possibility of modelling background flows with isolated regions of vorticity is rigorously established.

1. **Introduction.** Tsunami waves are gravity water waves mostly generated by undersea earthquakes, cf. [2], which cause a vertical displacement of the entire column of water above the fault region, thus giving the tsunami its initial wave profile. The waves then propagate over large distances without essentially changing their shape, a characteristic feature observable for example in the May 1960 tsunami that set out off the coast of Chile and travelled almost $17000km$ across the Pacific Ocean until it hit Japan (cf. [8, 21, 22]). Tsunami waves travel at very high, almost constant speed and their wave length is typically hundreds of kilometers long whereas their amplitude is relatively small (about $0.5m$, cf. [21]). While out in the open ocean, where the water depth is relatively uniform (eg. the Central Pacific Basin is approximately $4.3km$ deep, cf. [8]), the evolution of a tsunami is essentially governed by linear theory, the typical wave speed being $\sqrt{gh}$, where $g$ is the gravitational acceleration and $h$ the average water depth, cf. [11]. When a tsunami approaches the shoreline, the front of the wave slows down as the depth decreases, causing the water to pile up vertically near the coast since the back of the wave is still out in the open ocean travelling at very high speed. The resulting damage by surging water and inundation is often far more devastating than the effects of the preceding earthquake itself. In the case of the tsunami that hit Japan on March 11, 2011, an undersea megathrust earthquake of magnitude 9 occurred in the region where the pacific plate is subducting under the plate beneath the Japanese island of Honshu, as reported by the U.S. Geological Survey. When the stresses that had been building up in this process were finally released, the break caused the sea floor to rise by several meters in a rupture zone $300km$ long and $150km$ wide, with the epicenter about $70km$ off the coast of the island of Honshu. The earthquake resulted in a major tsunami which devastated entire towns along the pacific coast of northern

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