J. Math. Fluid Mech. 14 (2012), 141–158

1422-6928/12/010141-18
DOI 10.1007/s00021-011-0055-0

Journal of Mathematical Fluid Mechanics

On Some Background Flows for Tsunami Waves

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Communicated by A. Constantin

Abstract. With the aim to describe the state of the sea in a coastal region prior to the arrival of a tsunami, we show the existence of background flow fields with a flat free surface which model isolated regions of vorticity outside of which the water is at rest.

Mathematics Subject Classification (2010). 35Q31, 76B15, 35J61, 37N11.

Keywords. Tsunami background state, Isolated vorticity region.

1. Introduction

Tsunami waves are generated by a sudden vertical displacement of a body of water on a massive scale, caused by landslides, volcanic eruptions or, most commonly, by undersea earthquakes [2]. Tectonic collisions in the form of thrust (or normal) faults sometimes make the ocean floor rise (or drop) by a few meters, causing the column of water directly above to rise (or fall) as well and thereby creating an initial wave profile of elevation (or of depression), as it was the case with the December 2004 tsunami cf. [4,5,9,10,16,23]. Tsunami waves are a special type of gravity water waves, with typical wavelength of hundreds of kilometers. They can travel over thousands of kilometers at very high speed with little loss of energy, a spectacular example being the May 1960 tsunami that originated near the Chilean coast (due to the largest earthquake ever recorded) and propagated across the Pacific Ocean devastating coastal areas in Hawaii and Japan, 10,000 km respectively 17,000 km far from the Chilean coast [8,24]. Away from the shore, where the ocean can be assumed to have uniform depth over large distances (e.g. the ocean floor of the Central Pacific Basin is relatively uniform, with a mean water depth of about 4,300 m cf. [8]), the evolution of the wave is governed essentially by linear water wave theory, the typical wave speed being $\sqrt{gh}$ with g the gravitational constant of acceleration and h the average depth of the sea [11,23]. The amplitude of a tsunami wave out in the open sea is typically very small (roughly about 0.5 m cf. [23]), but when it approaches a gently sloping beach, the front of the wave slows down causing the water to pile up vertically since the back of the wave is still hundreds of kilometers out in the sea, travelling at much higher speed. The enormous amounts of water involved in this process, account for much of the devastating effects tsunami waves have in coastal areas.

Before the arrival of the tsunami waves at the shore, the water in that region is unlikely to be still: even in the presence of surface waves of small amplitude or for a flat free surface, beneath the surface there could be considerable motion due to the presence of currents (already for irrotational flows with a free surface, an underlying uniform current complicates considerably the dynamics of the flow since without a current all particle paths describe a non-closed loop [3] whereas certain currents can produce closed particle paths [14]). Taking into account currents, it seems essential in a reasonable model for tsunami waves to allow for some kind of background flow field, which models the motion of water in the absence of waves. While most investigations are restricted to irrotational flows which model background states