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Singular solutions for a class of traveling wave equations arising in hydrodynamics

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A B S T R A C T

We give an exhaustive characterization of singular weak solutions for ordinary differential equations of the form $\ddot{u}\,u + \frac{1}{2}\dot{u}^2 + F'(u) = 0$, where F is an analytic function. Our motivation stems from the fact that in the context of hydrodynamics several prominent equations are reducible to an equation of this form upon passing to a moving frame. We construct peaked and cusped waves, fronts with finite-time decay and compact solitary waves. We prove that one cannot obtain peaked and compactly supported traveling waves for the same equation. In particular, a peaked traveling wave cannot have compact support and vice versa. To exemplify the approach we apply our results to the Camassa–Holm equation and the equation for surface waves of moderate amplitude, and show how the different types of singular solutions can be obtained varying the energy level of the corresponding planar Hamiltonian systems.

1. Introduction

In the present paper we propose to study certain types of weak solutions for ordinary differential equations (ODE) of the form

$$\ddot{u}\,u + \frac{1}{2}\dot{u}^2 + F'(u) = 0, \tag{1}$$

where F is an analytic function. Our motivation stems from the fact that a variety of model equations arising in the context of hydrodynamics, among them the well-known Camassa–Holm equation (cf. [1–3]) and the related equation for surface waves of moderate amplitude (cf. [4–8]), are reducible to an ODE of the form (1) upon passing to a moving frame. Owing to the fact that every solution of Eq. (1) may be interpreted as

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