LIMIT CYCLES OF CONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS FORMED BY LINEAR AND QUADRATIC ISOCHRONOUS CENTERS II

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Abstract. We study the crossing periodic orbits and limit cycles of the planar continuous piecewise differential systems separated by the straight-line $x = 0$ having in $x > 0$ the general quadratic isochronous center $\dot{x} = -y + x^2$, $\dot{y} = x(1 + y)$ after an affine transformation, and in $x < 0$ an arbitrary quadratic isochronous center except for the quadratic isochronous center $\dot{x} = -y + x^2 - y^2$, $\dot{y} = x(1 + 2y)$ which has been studied in [4]. For these continuous, piecewise differential systems the upper bound of crossing limit cycles is 2, and there are realized examples having one crossing limit cycle.

1. Introduction

In the qualitative theory of planar differential systems a *limit cycle* is an isolated periodic solution in the set of all periodic solutions, which remained the most sought solutions when modeling physical systems in the plane. As far as we known the notion of limit cycle appeared in the year 1885 in the work of Poincaré [13].

Most of the early examples in the theory of limit cycles in planar differential systems were commonly related to practical problems with mechanical and electronic systems, but periodic behavior appears in all branches of the sciences. To determine the existence or non–existence of limit cycles is one of the more difficult objects in the qualitative theory of planar differential equations. A large amount of references deals with the subject of limit cycles, many of them motivated for the famous Hilbert's 16th problem, see for details [5, 7, 8].

Since 1930's the study of the limit cycles also became important in the continuous and discontinuous piecewise differential systems separated by a straight line, due to their applications to mechanics, electrical circuits, ... see for instance the books [1, 2, 14] and the references therein.

As usual a *center* p of a planar differential system is a singular point for which there is a neighborhood U such that $U \setminus \{p\}$ is filled with periodic orbits. When all the periodic orbits surrounding a center have the same period this center is called *isochronous*. The centers started to be studied by Poincaré [12] and Dulac [3], but the notion of isochronocity goes back to Huygens [6] in 1673.

In this paper we consider continuous piecewise differential systems separated by the straight line $x = 0$ having in $x \leq 0$ and in $x \geq 0$ quadratic isochronous centers, and we want to study the non–existence, and the existence of crossing periodic orbits and of crossing limit cycles, and in this last case we want also to know the maximum number of crossing limit cycles for these systems.

Here a *crossing periodic orbit* or a *crossing limit cycle* is a periodic orbit or a limit cycle which intersects exactly in two points the line of separation $x = 0$.

The *continuity* of a piecewise differential system separated by the straight line $x = 0$ formed by two centers means that the two vector fields defined by these two quadratic systems with isochronous centbbers coincide on the line of separation $x = 0$. So a continuous piecewise differential system is a continuous differential system in $\mathbb{R}^2$ and is an analytic differential system in $\mathbb{R}^2 \setminus \{x = 0\}$.

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1.1. **Quadratic isochronous centers.** We consider the quadratic polynomial differential systems having an isochronous center. This kind of centers were classified by Loud in the paper [11]. Those systems after an affine change of coordinates become one of the following four systems:

$$\dot{x} = -y + x^2 - y^2, \qquad \dot{y} = x(1+2y), \tag{1}$$ {nle1}

$$\dot{x} = -y + x^2, \qquad \dot{y} = x(1+y), \tag{2}$$ {nle2}

$$\dot{x} = -y - \frac{4x^2}{3}, \qquad \dot{y} = x\left(1 - \frac{16}{3}y\right), \tag{3}$$ {nle3}

$$\dot{x} = -y + \frac{16}{3}x^2 - \frac{4}{3}y^2, \qquad \dot{y} = x\left(1 + \frac{8}{3}y\right). \tag{4}$$ {nle4}

We are interested in the general expressions of the quadratic isochronous centers. So we transform their normal forms (1), (2), (3) and (4) through the following general affine change of variables

$$(x,y) \to (a_1x + b_1y + c_1, a_2x + b_2y + c_2), \tag{5}$$ {achv}

with

$$a_1b_2 - a_2b_1 \neq 0. \tag{6}$$ {el}

Generalized isochronous system (1). Using the change of variables (5) the quadratic system (1) becomes

$$\begin{aligned}
\dot{x} = (&-b_2c_1^2 + b_1c_1 + 2b_1c_2c_1 + b_2c_2^2 + b_2c_2\\
&+ (2a_2b_1c_1 + 2a_1b_1c_2 - 2a_1b_2c_1 + 2a_2b_2c_2 + a_1b_1 + a_2b_2)\,x\\
&+ \left(2b_1^2c_2 + 2b_2^2c_2 + b_1^2 + b_2^2\right) y + \left(2a_2b_1^2 + 2a_2b_2^2\right) xy\\
&+ \left(a_1^2\,(-b_2) + 2a_2a_1b_1 + a_2^2b_2\right) x^2 + \left(b_2^3 + b_1^2b_2\right) y^2\big) / (a_2b_1 - a_1b_2),\\
\dot{y} = (&\,a_2c_1^2 - a_1c_1 - 2a_1c_2c_1 - a_2c_2^2 - a_2c_2 + \left(-2a_1^2c_2 - 2a_2^2c_2 - a_1^2 - a_2^2\right) x\\
&+ (2a_2b_1c_1 - 2a_1b_1c_2 - 2a_1b_2c_1 - 2a_2b_2c_2 - a_1b_1 - a_2b_2)\,y\\
&+ \left(-2a_1^2b_2 - 2a_2^2b_2\right) xy + \left(-a_2^3 - a_1^2a_2\right) x^2\\
&+ \left(a_2b_1^2 - 2a_1b_2b_1 - a_2b_2^2\right) y^2\big) / (a_2b_1 - a_1b_2).
\end{aligned} \tag{7}$$ {gnle1}

Since $(x^2+y^2)/(2y+1)$ is a first integral of system (1), doing to it the change of variables (5) we get the following first integral of the generalized isochronous quadratic system (7)

$$H_2(x,y) = \frac{(a_1x + b_1y + c_1)^2 + (a_2x + b_2y + c_2)^2}{2\,(a_2x + b_2y + c_2) + 1}.$$

Generalized isochronous system (2). System (2) is equivalent to the following generalized isochronous system after the linear change of variables (5)

$$\begin{aligned}
\dot{x} = (&\,b_2c_1^2 - b_1c_1 - b_1c_2c_1 - b_2c_2\\
&+ (-a_2b_1c_1 - a_1b_1c_2 + 2a_1b_2c_1 - a_1b_1 - a_2b_2)\,x\\
&+ \left(b_1^2\,(-c_2) + b_2b_1c_1 - b_1^2 - b_2^2\right) y + \left(a_1b_1b_2 - a_2b_1^2\right) xy\\
&+ \left(a_1^2b_2 - a_1a_2b_1\right) x^2\big) / (a_1b_2 - a_2b_1),\\
\dot{y} = (&\,a_2c_1^2 - a_1c_1 - a_1c_2c_1 - a_2c_2 + \left(a_1^2\,(-c_2) + a_2a_1c_1 - a_1^2 - a_2^2\right) x\\
&+ (2a_2b_1c_1 - a_1b_1c_2 - a_1b_2c_1 - a_1b_1 - a_2b_2)\,y\\
&+ \left(a_1a_2b_1 - a_1^2b_2\right) xy + \left(a_2b_1^2 - a_1b_1b_2\right) y^2\big) / (a_1b_2 - a_2b_1).
\end{aligned} \tag{8}$$ {gnle2}

The quadratic system (2) has the first integral $(x^2+y^2)/(y+1)^2$. Therefore a first integral of system (8) is

$$H_3(x,y) = \frac{(a_1x + b_1y + c_1)^2 + (a_2x + b_2y + c_2)^2}{(a_2x + b_2y + c_2 + 1)^2}.$$

Generalized isochronous system (3). The quadratic system (3) is equivalent to the following generalized quadratic system after the linear change of variables (5)

$$
\begin{aligned}
\dot{x} = &\big(4b_2c_1^2 + 3b_1c_1 - 16b_1c_2c_1 + 3b_2c_2 \\
&+ (-16a_2b_1c_1 - 16a_1b_1c_2 + 8a_1b_2c_1 + 3a_1b_1 + 3a_2b_2)\, x \\
&+ \left(-16b_1^2c_2 - 8b_2b_1c_1 + 3b_1^2 + 3b_2^2\right) y + \left(-16a_2b_1^2 - 8a_1b_2b_1\right) xy \\
&+ \left(4a_1^2b_2 - 16a_1a_2b_1\right) x^2 - 12b_1^2b_2y^2\big) /3\,(a_2b_1 - a_1b_2)\,, \\
\dot{y} = &\big(- 4a_2c_1^2 - 3a_1c_1 + 16a_1c_2c_1 - 3a_2c_2 \\
&+ \left(16a_1^2c_2 + 8a_2a_1c_1 - 3a_1^2 - 3a_2^2\right) x \\
&+ (-8a_2b_1c_1 + 16a_1b_1c_2 + 16a_1b_2c_1 - 3a_1b_1 - 3a_2b_2)\, y \\
&+ \left(16a_1^2b_2 + 8a_2a_1b_1\right) xy + 12a_1^2a_2x^2 \\
&+ \left(16a_1b_1b_2 - 4a_2b_1^2\right) y^2\big) /3\,(a_2b_1 - a_1b_2)\,.
\end{aligned} \tag{9}
$$

{gnle3}

Since $(32x^2 - 24y + 9)^2/(3 - 16y)$ is a first integral of system (9), then a first integral of system (9) is

$$
H_4(x,y) = \frac{\left(32\,(a_1x + b_1y + c_1)^{\,2} - 24\,(a_2x + b_2y + c_2) + 9\right)^{\,2}}{3 - 16\,(a_2x + b_2y + c_2)}.
$$

Generalized isochronous system (4). Doing the affine change of variables (5) the generalized isochronous system for the quadratic system (4) is

$$
\begin{aligned}
\dot{x} = &\big(- 16b_2c_1^2 + 3b_1c_1 + 8b_1c_2c_1 + 4b_2c_2^2 + 3b_2c_2 \\
&+ (8a_2b_1c_1 + 8a_1b_1c_2 - 32a_1b_2c_1 + 8a_2b_2c_2 + 3a_1b_1 + 3a_2b_2)\, x \\
&+ \left(8b_1^2c_2 - 24b_2b_1c_1 + 8b_2^2c_2 + 3b_1^2 + 3b_2^2\right) y \\
&+ \left(8a_2b_1^2 - 24a_1b_2b_1 + 8a_2b_2^2\right) xy + \left(-16a_1^2b_2 + 8a_2a_1b_1 + 4a_2^2b_2\right) x^2 \\
&+ \left(4b_2^3 - 8b_1^2b_2\right) y^2\big) /3\,(a_2b_1 - a_1b_2)\,, \\
\dot{y} = &\big(16a_2c_1^2 - 3a_1c_1 - 8a_1c_2c_1 - 4a_2c_2^2 - 3a_2c_2 \\
&+ \left(-8a_1^2c_2 + 24a_2a_1c_1 - 8a_2^2c_2 - 3a_1^2 - 3a_2^2\right) x \\
&+ (32a_2b_1c_1 - 8a_1b_1c_2 - 8a_1b_2c_1 - 8a_2b_2c_2 - 3a_1b_1 - 3a_2b_2)\, y \\
&+ \left(-8a_1^2b_2 + 24a_2a_1b_1 - 8a_2^2b_2\right) xy + \left(8a_1^2a_2 - 4a_2^3\right) x^2 \\
&+ \left(16a_2b_1^2 - 8a_1b_2b_1 - 4a_2b_2^2\right) y^2\big) /3\,(a_2b_1 - a_1b_2)\,.
\end{aligned} \tag{10}
$$

{gnle4}

The quadratic system (4) has the first integral $(-256x^2 + 128y^2 + 96y + 9)/(8y + 3)^4$, which gives the following first integral for system (10)

$$
H_5(x,y) = \frac{-256\,(a_1x + b_1y + c_1)^{\,2} + 128\,(a_2x + b_2y + c_2)^2 + 96\,(a_2x + b_2y + c_2) + 9}{(8\,(a_2x + b_2y + c_2) + 3)^4}.
$$

1.2. **Statement of the main results.** In what follows we characterize the existence and non-existence of crossing periodic orbits and crossing limit cycles for continuous piecewise linear differential systems separated by one straight line formed by two quadratic isochronous centers.

{thm2}

Theorem 1. *The following statements hold for the continuous piecewise differential systems formed by two generalized isochronous quadratic centers separated by the straight line $x = 0$.*

(a) If the generalized centers are (1) *and* (1)*, then the piecewise differential systems can have crossing periodic orbits but they cannot have crossing limit cycles.*

(b) If the generalized centers are (1) *and* (2)*, then the piecewise differential systems can have crossing periodic orbits but they cannot have crossing limit cycles.*

(c) If the generalized centers are (1) *and* (3) *then the piecewise differential systems can have at most one limit cycle.*

(d) *If the generalized centers are* (1) *and* (4) *then the piecewise differential systems can have at most one limit cycle.*

(e) *If the generalized centers are* (2) *and* (2) *then the piecewise differential systems have no crossing periodic orbits, and consequently no crossing limit cycles.*

(f) *If the generalized centers are* (2) *and* (3) *then the piecewise differential systems have no limit cycles.*

(g) *If the generalized centers are* (2) *and* (4) *then the piecewise differential systems can have at most two limit cycles, and there are systems in this case with exactly one limit cycle.*

The first four statement of Theorem 1 were proved in [4], here we prove all the other statements of Theorem 1.

2. Proof of Theorem 1 {s3}

In what follows we consider a continuous piecewise differential system formed by two generalized isochronous systems (i) and (j) separated by the straight line $x = 0$, where $i, j \in \{8, 9, 10\}$. In the first generalized systems (i) we rename the parameters a_2, b_2 and c_2 by α_1, β_1 and γ_1, respectively; and in the second generalized systems (j) we rename the parameters a_1, b_1, c_1, a_2, b_2 and c_2 by a_2, b_2, c_2, α_2, β_2 and γ_2, respectively. Doing this condition (6) becomes

$$\alpha_1 b_1 - a_1\beta_1 \neq 0 \quad \text{and} \quad \alpha_2 b_2 - a_2\beta_2 \neq 0, \tag{11}$$ {e11}

for system (i) and (j) respectively.

It the piecewise differential system (i)-(j) has a crossing periodic orbit intersecting the line of separation in the points $(0, y_i)$ for $i = 1, 2$, these two points must satisfy the following algebraic system

$$H_i(0, y_1) - H_i(0, y_2) = 0, \qquad H_j(0, y_1) - H_j(0, y_2) = 0, \tag{12}$$ {eij}

where H_i and H_j are first integrals of the generalized systems (i) and (j).

Proof of statement (e) *of Theorem* 1. Consider the continuous piecewise differential system formed by two generalized isochronous quadratic systems (8) separated by the straight line $x = 0$. Before getting the conditions for which the piecewise differential systems formed by these two systems (8) be continuous we provide an upper bound for the maximum number of limit cycles which these systems can exhibit. Then we must solve equations (12) for finding this upper boound. For our piecewise differential system equations (12) become

$$\begin{aligned}
&(y_1 - y_2)(2b_1c_1\gamma_1^2 + 4b_1c_1\gamma_1 - 2b_1\beta_1^2c_1y_1y_2 + 2\beta_1\gamma_1^2 + 2\beta_1\gamma_1 + 2b_1c_1 + 2b_1^2\beta_1\gamma_1y_1y_2\\
&+2b_1^2\beta_1y_1y_2 + b_1^2\gamma_1^2y_1 + b_1^2\gamma_1^2y_2 + 2b_1^2\gamma_1y_1 + 2b_1^2\gamma_1y_2 + b_1^2y_1 + b_1^2y_2 - 2\beta_1c_1^2\gamma_1 - 2\beta_1c_1^2\\
&-\beta_1^2c_1^2y_1 - \beta_1^2c_1^2y_2 + 2\beta_1^2\gamma_1y_1 + 2\beta_1^2\gamma_1y_2 + 2\beta_1^3y_1y_2 + \beta_1^2y_1 + \beta_1^2y_2)/[(\gamma_1 + \beta_1y_1 + 1)^2\\
&(\gamma_1 + \beta_1y_2 + 1)^2] = 0,\\
&(y_1 - y_2)(2b_2c_2\gamma_2^2 + 4b_2c_2\gamma_2 - 2b_2\beta_2^2c_2y_1y_2 + 2\beta_2\gamma_2^2 + 2\beta_2\gamma_2 + 2b_2c_2 + 2b_2^2\beta_2\gamma_2y_1y_2\\
&+2b_2^2\beta_2y_1y_2 + b_2^2\gamma_2^2y_1 + b_2^2\gamma_2^2y_2 + 2b_2^2\gamma_2y_1 + 2b_2^2\gamma_2y_2 + b_2^2y_1 + b_2^2y_2 - 2\beta_2c_2^2\gamma_2 - 2\beta_2c_2^2\\
&-\beta_2^2c_2^2y_1 - \beta_2^2c_2^2y_2 + 2\beta_2^2\gamma_2y_1 + 2\beta_2^2\gamma_2y_2 + 2\beta_2^3y_1y_2 + \beta_2^2y_1 + \beta_2^2y_2)/[(\gamma_2 + \beta_2y_1 + 1)^2\\
&(\gamma_2 + \beta_2y_2 + 1)^2] = 0,
\end{aligned} \tag{13}$$ {e4}

solving the first equation of (13) with respect to y_1 we obtain

$$y_1 = \frac{-2b_1c_1(\gamma_1+1)^2 - b_1^2(\gamma_1+1)^2y_2 + \beta_1(c_1^2(2\gamma_1 + \beta_1y_2 + 2) - 2\gamma_1(\gamma_1+1) - y_2(2\beta_1\gamma_1 + \beta_1))}{-2\beta_1^2b_1c_1y_2 + b_1^2(\gamma_1+1)(\gamma_1 + 2\beta_1y_2 + 1) + \beta_1^2(-c_1^2 + 2\gamma_1 + 2\beta_1y_2 + 1)}.$$

Replacing this value of y_1 into the second equation of (13) and solving it with respect to y_2, we get a unique solution solution excluding the trivial one $y_1 = y_2$. Replacing this value of y_2 into the expression of y_1, we conclude that we have at most 1 pair (y_1, y_2) given by

$$
\begin{aligned}
y_1 = {}& [-b_2c_2\gamma_2^2\beta_1^3 - \beta_2\gamma_2^2\beta_1^3 - b_2c_2\beta_1^3 + c_2^2\beta_2\beta_1^3 - 2b_2c_2\gamma_2\beta_1^3 + c_2^2\beta_2\gamma_2\beta_1^3 - \beta_2\gamma_2\beta_1^3\\
&+b_1b_2c_1c_2\gamma_2^2\beta_1^2 + b_1c_1\beta_2\gamma_2^2\beta_1^2 + b_1b_2c_1c_2\beta_1^2 - b_1c_1c_2^2\beta_2\beta_1^2 + 2b_1b_2c_1c_2\gamma_2\beta_1^2 - b_1c_1c_2^2\beta_2\gamma_2\beta_1^2\\
&+b_1c_1\beta_2\gamma_2\beta_1^2 - c_1^2\beta_2^3\beta_1 + b_2c_1^2c_2\beta_2^2\beta_1 + \beta_2^3\gamma_1^2\beta_1 - b_2c_2\beta_2^2\gamma_1^2\beta_1 + b_2^2\beta_2\gamma_1^2\beta_1 - b_1^2b_2c_2\gamma_2^2\beta_1\\
&-b_1^2\beta_2\gamma_2^2\beta_1 - b_1^2b_2c_2\gamma_1\gamma_2^2\beta_1 - b_1^2\beta_2\gamma_1\gamma_2^2\beta_1 - b_1^2b_2c_2\beta_1 - b_2^2c_1^2\beta_2\beta_1 + b_1^2c_2^2\beta_2\beta_1 - c_1^2\beta_2^3\gamma_1\beta_1\\
&+\beta_2^3\gamma_1\beta_1 + b_2c_1^2c_2\beta_2^2\gamma_1\beta_1 - b_2c_2\beta_2^2\gamma_1\beta_1 - b_1^2b_2c_2\gamma_1\beta_1 + b_2^2\beta_2\gamma_1\beta_1 - b_2^2c_1^2\beta_2\gamma_1\beta_1 + b_1^2c_2^2\beta_2\gamma_1\beta_1\\
&+b_2^2\beta_2\gamma_1^2\gamma_2\beta_1 - 2b_1^2b_2c_2\gamma_2\beta_1 - b_1^2\beta_2\gamma_2\beta_1 - b_2^2c_1^2\beta_2\gamma_2\beta_1 + b_1^2c_2^2\beta_2\gamma_2\beta_1 - 2b_1^2b_2c_2\gamma_1\gamma_2\beta_1\\
&-b_1^2\beta_2\gamma_1\gamma_2\beta_1 + b_2^2\beta_2\gamma_1\gamma_2\beta_1 - b_2^2c_1^2\beta_2\gamma_1\gamma_2\beta_1 + b_1^2c_2^2\beta_2\gamma_1\gamma_2\beta_1 + b_1c_1\beta_2^3 - b_1b_2c_1c_2\beta_2^2 + b_1c_1\beta_2^3\gamma_1^2\\
&-b_1b_2c_1c_2\beta_2^2\gamma_1^2 + b_1b_2^2c_1\beta_2\gamma_1^2 + [(\beta_2(\gamma_1+1) - \beta_1(\gamma_2+1))((\gamma_1+1)(\gamma_2+1)b_1^2 - c_1(\gamma_2\beta_1 + \beta_1\\
&+\beta_2 + \beta_2\gamma_1)b_1 + \beta_1(\beta_2c_1^2 + \beta_1 - \beta_2\gamma_1 + \beta_1\gamma_2))((\gamma_1+1)(\gamma_2+1)b_2^2 - c_2(\gamma_2\beta_1 + \beta_1 + \beta_2\\
&+\beta_2\gamma_1)b_2 + \beta_2(\beta_1c_2^2 + \beta_2 + \beta_2\gamma_1 - \beta_1\gamma_2))((\gamma_1+1)(b_2c_2(\gamma_2+1) + \beta_2(\gamma_2 - c_2^2))b_1^2\\
&-c_1((\gamma_1+1)(\gamma_2+1)b_2^2 + c_2(\beta_1(\gamma_2+1) - \beta_2(\gamma_1+1))b_2 + \beta_2(-c_2^2\beta_1 + \beta_2 + \beta_2\gamma_1 + \beta_1\gamma_2))b_1\\
&+\beta_1((c_1^2 - \gamma_1)(\gamma_2+1)b_2^2 + c_2(-\beta_2c_1^2 + \beta_1 + \beta_2\gamma_1 + \beta_1\gamma_2)b_2 + \beta_2(\beta_2c_1^2 - c_2^2\beta_1 - \beta_2\gamma_1\\
&+\beta_1\gamma_2)))]^{1/2} + b_1b_2^2c_1\beta_2 + 2b_1c_1\beta_2^3\gamma_1 - 2b_1b_2c_1c_2\beta_2^2\gamma_1 + 2b_1b_2^2c_1\beta_2\gamma_1 + b_1b_2^2c_1\beta_2\gamma_1^2\gamma_2\\
&+b_1b_2^2c_1\beta_2\gamma_2 + 2b_1b_2^2c_1\beta_2\gamma_1\gamma_2]/[b_1^2(-(\gamma_1+1))(b_2^2(-(\gamma_2+1))(\beta_1(\gamma_2+1) - \beta_2(\gamma_1+1))\\
&-b_2\beta_2^2c_2(\gamma_1+1) + \beta_2^2(\beta_2(\gamma_1+1) + \beta_1(c_2^2 - 2\gamma_2 - 1))) - b_1\beta_1^2c_1(b_2^2(\gamma_2+1)^2\\
&+\beta_2^2(-c_2^2 + 2\gamma_2 + 1)) + \beta_1^2(b_2^2(\gamma_2+1)(\beta_1(\gamma_2+1) + \beta_2(c_1^2 - 2\gamma_1 - 1)) + \beta_2^2b_2c_2(-c_1^2 + 2\gamma_1 + 1)\\
&+\beta_2^2(\beta_2(c_1^2 - 2\gamma_1 - 1) + \beta_1(-c_2^2 + 2\gamma_2 + 1)))]
\end{aligned}
\tag{14}
$$

{e5}

and

$$
\begin{aligned}
y_2 = {}& -[b_2c_2\gamma_2^2\beta_1^3 + \beta_2\gamma_2^2\beta_1^3 + b_2c_2\beta_1^3 - c_2^2\beta_2\beta_1^3 + 2b_2c_2\gamma_2\beta_1^3 - c_2^2\beta_2\gamma_2\beta_1^3 + \beta_2\gamma_2\beta_1^3\\
&-b_1b_2c_1c_2\gamma_2^2\beta_1^2 - b_1c_1\beta_2\gamma_2^2\beta_1^2 - b_1b_2c_1c_2\beta_1^2 + b_1c_1c_2^2\beta_2\beta_1^2 - 2b_1b_2c_1c_2\gamma_2\beta_1^2 + b_1c_1c_2^2\beta_2\gamma_2\beta_1^2\\
&-b_1c_1\beta_2\gamma_2\beta_1^2 + c_1^2\beta_2^3\beta_1 - b_2c_1^2c_2\beta_2^2\beta_1 - \beta_2^3\gamma_1^2\beta_1 + b_2c_2\beta_2^2\gamma_1^2\beta_1 - b_2^2\beta_2\gamma_1^2\beta_1 + b_1^2b_2c_2\gamma_2^2\beta_1\\
&+b_1^2\beta_2\gamma_2^2\beta_1 + b_1^2b_2c_2\gamma_1\gamma_2^2\beta_1 + b_1^2\beta_2\gamma_1\gamma_2^2\beta_1 + b_1^2b_2c_2\beta_1 + b_2^2c_1^2\beta_2\beta_1 - b_1^2c_2^2\beta_2\beta_1 + c_1^2\beta_2^3\gamma_1\beta_1\\
&-\beta_2^3\gamma_1\beta_1 - b_2c_1^2c_2\beta_2^2\gamma_1\beta_1 + b_2c_2\beta_2^2\gamma_1\beta_1 + b_1^2b_2c_2\gamma_1\beta_1 - b_2^2\beta_2\gamma_1\beta_1 + b_2^2c_1^2\beta_2\gamma_1\beta_1 - b_1^2c_2^2\beta_2\gamma_1\beta_1\\
&-b_2^2\beta_2\gamma_1^2\gamma_2\beta_1 + 2b_1^2b_2c_2\gamma_2\beta_1 + b_1^2\beta_2\gamma_2\beta_1 + b_2^2c_1^2\beta_2\gamma_2\beta_1 - b_1^2c_2^2\beta_2\gamma_2\beta_1 + 2b_1^2b_2c_2\gamma_1\gamma_2\beta_1\\
&+b_1^2\beta_2\gamma_1\gamma_2\beta_1 - b_2^2\beta_2\gamma_1\gamma_2\beta_1 + b_2^2c_1^2\beta_2\gamma_1\gamma_2\beta_1 - b_1^2c_2^2\beta_2\gamma_1\gamma_2\beta_1 - b_1c_1\beta_2^3 + b_1b_2c_1c_2\beta_2^2 - b_1c_1\beta_2^3\gamma_1^2\\
&+b_1b_2c_1c_2\beta_2^2\gamma_1^2 - b_1b_2^2c_1\beta_2\gamma_1^2 + [(\beta_2(\gamma_1+1) - \beta_1(\gamma_2+1))((\gamma_1+1)(\gamma_2+1)b_1^2\\
&-c_1(\gamma_2\beta_1 + \beta_1 + \beta_2 + \beta_2\gamma_1)b_1 + \beta_1(\beta_2c_1^2 + \beta_1 - \beta_2\gamma_1 + \beta_1\gamma_2))((\gamma_1+1)(\gamma_2+1)b_2^2\\
&-c_2(\gamma_2\beta_1 + \beta_1 + \beta_2 + \beta_2\gamma_1)b_2 + \beta_2(\beta_1c_2^2 + \beta_2 + \beta_2\gamma_1 - \beta_1\gamma_2))((\gamma_1+1)(b_2c_2(\gamma_2+1)\\
&+\beta_2(\gamma_2 - c_2^2))b_1^2 - c_1((\gamma_1+1)(\gamma_2+1)b_2^2 + c_2(\beta_1(\gamma_2+1) - \beta_2(\gamma_1+1))b_2 + \beta_2(-c_2^2\beta_1 + \beta_2\\
&+\beta_2\gamma_1 + \beta_1\gamma_2))b_1 + \beta_1((c_1^2 - \gamma_1)(\gamma_2+1)b_2^2 + c_2(-\beta_2c_1^2 + \beta_1 + \beta_2\gamma_1 + \beta_1\gamma_2)b_2 + \beta_2(\beta_2c_1^2\\
&-c_2^2\beta_1 - \beta_2\gamma_1 + \beta_1\gamma_2)))]^{1/2} - b_1b_2^2c_1\beta_2 - 2b_1c_1\beta_2^3\gamma_1 + 2b_1b_2c_1c_2\beta_2^2\gamma_1 - 2b_1b_2^2c_1\beta_2\gamma_1\\
&-b_1b_2^2c_1\beta_2\gamma_1^2\gamma_2 - b_1b_2^2c_1\beta_2\gamma_2 - 2b_1b_2^2c_1\beta_2\gamma_1\gamma_2]/[b_1^2(-(\gamma_1+1))(b_2^2(-(\gamma_2+1))(\beta_1(\gamma_2+1)\\
&-\beta_2(\gamma_1+1)) - b_2\beta_2^2c_2(\gamma_1+1) + \beta_2^2(\beta_2(\gamma_1+1) + \beta_1(c_2^2 - 2\gamma_2 - 1))) - b_1\beta_1^2c_1(b_2^2(\gamma_2+1)^2\\
&+\beta_2^2(-c_2^2 + 2\gamma_2 + 1)) + \beta_1^2(b_2^2(\gamma_2+1)(\beta_1(\gamma_2+1) + \beta_2(c_1^2 - 2\gamma_1 - 1)) + \beta_2^2b_2c_2(-c_1^2 + 2\gamma_1 + 1)\\
&+\beta_2^2(\beta_2(c_1^2 - 2\gamma_1 - 1) + \beta_1(-c_2^2 + 2\gamma_2 + 1)))].
\end{aligned}
\tag{15}
$$

{e6}

In summary the piecewise differential systems formed by two systems (8) can have at most one limit cycle. In addition, if we calculate $(y_1 - y_2)^2/4$ from (14) and (15) we get

$$
\begin{aligned}
(y_1 - y_2)^2/4 = {}& (\beta_2(\gamma_1+1) - \beta_1(\gamma_2+1))(-b_1c_1(\beta_1\gamma_2 + \beta_2\gamma_1 + \beta_1 + \beta_2) + b_1^2(\gamma_1+1)(\gamma_2+1)\\
&+\beta_1(-\beta_2\gamma_1 + \beta_1\gamma_2 + \beta_1 + \beta_2c_1^2))(-b_2c_2(\beta_1\gamma_2 + \beta_2\gamma_1 + \beta_1 + \beta_2) + b_2^2(\gamma_1+1)(\gamma_2+1)\\
&+\beta_2(\beta_2\gamma_1 - \beta_1\gamma_2 + \beta_2 + \beta_1c_2^2))(b_1^2(\gamma_1+1)(b_2c_2(\gamma_2+1) + \beta_2(\gamma_2 - c_2^2)) - b_1c_1(b_2c_2(\beta_1(\gamma_2+1)\\
&-\beta_2(\gamma_1+1)) + b_2^2(\gamma_1+1)(\gamma_2+1) + \beta_2(\beta_2\gamma_1 + \beta_1\gamma_2 + \beta_2 + \beta_1(-c_2^2))) + \beta_1(b_2c_2(\beta_2\gamma_1 + \beta_1\gamma_2\\
&+\beta_1 - \beta_2c_1^2) + b_2^2(\gamma_2+1)(c_1^2 - \gamma_1) + \beta_2(-\beta_2\gamma_1 + \beta_1\gamma_2 + \beta_2c_1^2 - \beta_1c_2^2))).
\end{aligned}
\tag{16}
$$

{e7}

Of course, if the value of $(y_1 - y_2)^2/4 = 0$, then there are no crossing periodic orbtis. But,we will prove that if the piecewise differential systems formed by two systems (8) is continuous then $(y_1 - y_2)^2/4 = 0$, which proves that there are no crossing periodic orbits and consequently no limit cycles in this case.

Now in order that the piecewise differential system formed by two systems (8) be continuous they must coincide on $x=0$; i.e. the coefficients of the system must satisfy the following algebraic system

$$
\begin{aligned}
&-a_2b_1\beta_2\gamma_1c_1+a_1b_2\beta_1c_2\gamma_2-a_2b_1\beta_2c_1+a_1b_2\beta_1c_2-a_2\beta_1\beta_2\gamma_1+a_1\beta_1\beta_2\gamma_2+a_2\beta_1\beta_2c_1^2\\
&-a_1\beta_1\beta_2c_2^2+\alpha_2b_2\beta_1\gamma_1-\alpha_1b_1\beta_2\gamma_2+\alpha_2(-b_2)\beta_1c_1^2+\alpha_1b_1\beta_2c_2^2+\alpha_2b_1b_2\gamma_1c_1-\alpha_1b_1b_2c_2\gamma_2\\
&+\alpha_2b_1b_2c_1-\alpha_1b_1b_2c_2=0,\\[4pt]
&-a_2\beta_2b_1^2\gamma_1+a_1b_2^2\beta_1\gamma_2-a_2\beta_2b_1^2+a_1b_2^2\beta_1+a_2\beta_1\beta_2b_1c_1-a_1b_2\beta_1\beta_2c_2+a_1\beta_1\beta_2^2-a_2\beta_1^2\beta_2\\
&-\alpha_1\beta_2^2b_1+\alpha_2b_2\beta_1^2+\alpha_2b_2b_1^2\gamma_1-\alpha_1b_2^2b_1\gamma_2+\alpha_2b_2b_1^2-\alpha_1b_1b_2^2-\alpha_2b_2\beta_1b_1c_1+\alpha_1b_2\beta_2b_1c_2=0,\\[4pt]
&a_2\alpha_1\beta_2\gamma_1-a_1\alpha_2\beta_1\gamma_2-a_1\alpha_2b_2\gamma_1c_1+a_2\alpha_1b_1c_2\gamma_2-a_1\alpha_2b_2c_1+a_2\alpha_1b_1c_2-a_2\alpha_1\beta_2c_1^2\\
&+a_1\alpha_2\beta_1c_2^2+a_1a_2\beta_2\gamma_1c_1-a_1a_2\beta_1c_2\gamma_2+a_1a_2\beta_2c_1-a_1a_2\beta_1c_2-\alpha_1\alpha_2b_2\gamma_1+\alpha_1\alpha_2b_1\gamma_2\\
&+\alpha_1\alpha_2b_2c_1^2-\alpha_1\alpha_2b_1c_2^2=0,\\[4pt]
&a_2\alpha_1\beta_1\beta_2-a_1\alpha_2\beta_1\beta_2+a_2\alpha_1b_1b_2\gamma_2-a_1\alpha_2b_1b_2\gamma_1+a_2\alpha_1b_1b_2-a_1\alpha_2b_1b_2+a_1a_2b_1\beta_2\gamma_1\\
&-a_1a_2b_2\beta_1\gamma_2-a_1a_2b_2\beta_1+a_1a_2b_1\beta_2-2a_2\alpha_1b_1\beta_2c_1+a_2\alpha_1b_1\beta_2c_2-a_1\alpha_2b_2\beta_1c_1\\
&+2a_1\alpha_2b_2\beta_1c_2+a_1a_2\beta_1\beta_2c_1-a_1a_2\beta_1\beta_2c_2-\alpha_2\alpha_1b_2\beta_1+\alpha_2\alpha_1b_1\beta_2+2\alpha_2\alpha_1b_1b_2c_1\\
&-2\alpha_2\alpha_1b_1b_2c_2=0,\\[4pt]
&b_1-b_2=0,
\end{aligned}
\tag{17}
$$

{cfpc.2.2}

together with conditions (11).

We can easily see from the last equation that $b_2=b_1$. Solving the second equation of (17) we get

$$
\begin{aligned}
a_1=[&b_1^2(\beta_2(a_2\gamma_1+a_2-\alpha_1c_2)+\alpha_2\beta_1c_1)-b_1(\alpha_2\beta_1^2+\beta_2(a_2\beta_1c_1-\alpha_1\beta_2))+a_2\beta_1^2\beta_2\\
&+b_1^3(\alpha_1(\gamma_2+1)-\alpha_2(\gamma_1+1))]/[\beta_1(-b_1\beta_2c_2+b_1^2(\gamma_2+1)+\beta_2^2)],
\end{aligned}
\tag{18}
$$

{B1}

except when denominator $\beta_1\left(-b_1\beta_2c_2+b_1^2\left(\gamma_2+1\right)+\beta_2^2\right)=0$. This denominator vanishes if and only if one of the following three conditions holds:

$$
\begin{aligned}
d_1&=\{\beta_1=0\},\\
d_2&=\{\gamma_2=(\beta_2b_1c_2-b_1^2-\beta_2^2)/b_1^2\},\\
d_3&=\{b_1=0,\ \beta_2=0\}.
\end{aligned}
\tag{19}
$$

{d5}

Since $b_2=b_1$ then d_3 of (19) gives a contradiction with (11).

To continue solving (17) we must discuss all cases when $\beta_1(-b_1\beta_2c_2+b_1^2(\gamma_2+1)+\beta_2^2)=0$ and $\beta_1(-b_1\beta_2c_2+b_1^2(\gamma_2+1)+\beta_2^2)\neq 0$.

Case 1: Assume that $\beta_1\left(-b_1\beta_2c_2+b_1^2\left(\gamma_2+1\right)+\beta_2^2\right)\neq 0$. Using a_1 given by (18) we solve the fourth equation of the algebraic system (17) and we obtain the solutions

$$
\begin{aligned}
u_1&=\{\gamma_1=-(-\beta_1b_1c_1+b_1^2+\beta_1^2)/b_1^2\},\\
u_2&=\{\gamma_2=[b_1\left(a_2\left(\beta_1-\beta_2\left(\gamma_1+1\right)\right)+\alpha_2\beta_1c_1-2\alpha_2\beta_1c_2+\alpha_1\beta_2c_2\right)\\
&\qquad+\beta_2\left(\alpha_2\beta_1-\alpha_1\beta_2+a_2\beta_1\left(c_2-c_1\right)\right)+b_1^2\left(\alpha_2\gamma_1-\alpha_1+\alpha_2\right)]/[b_1\left(\alpha_1b_1-a_2\beta_1\right)]\},\\
u_3&=\{b_1=0,\ c_1=(\alpha_2\beta_1-\alpha_1\beta_2)/(a_2\beta_1)+c_2\},\\
u_4&=\{b_1=0,\ \beta_1=0\},\\
u_5&=\{b_1=0,\ \beta_2=0\},\\
u_6&=\{\alpha_1=a_2\beta_1/b_1,\ \gamma_1=-(\beta_1b_1\left(c_1-2c_2\right)+b_1^2+\beta_1\beta_2)/b_1^2\},\\
u_7&=\{\alpha_1=a_2\beta_1/b_1,\ \alpha_2=a_2\beta_2/b_1\},\\
u_8&=\{a_2=0,\ \alpha_1=0,\ \gamma_1=-(\beta_1b_1\left(c_1-2c_2\right)+b_1^2+\beta_1\beta_2)/b_1^2\},\\
u_9&=\{a_2=0,\ b_1=0,\ \alpha_1=\alpha_2\beta_1/\beta_2\}\quad\text{and}\\
u_{10}&=\{b_1=0,\ \alpha_1=0,\ \beta_1=0\}.
\end{aligned}
\tag{20}
$$

{u4}

The allowed solutions which do not contradict (11) are u_2, u_3, u_6 and u_8. Then we must discuss all these cases.

Subcase 1.1: Consider the set u_2 of (20). Solving the first equation of (17) we obtain one of the following sets of real solutions

$$
\begin{aligned}
v_1 &= \{\alpha_1 = [b_1^4(\gamma_1+1)(\alpha_2(\beta_2\gamma_1-\beta_1\gamma_1+\beta_2)-a_2\beta_2(\gamma_1+1)c_2+a_2\beta_2c_1(\gamma_1+1)\\
&\quad -2\alpha_2\beta_1c_2^2+2\alpha_2\beta_1c_1c_2)+b_1^3(a_2(\beta_1-\beta_2)\beta_2(\gamma_1+1)^2+\beta_1c_1(-\alpha_2\\
&\quad ((\beta_1+\beta_2)\gamma_1+\beta_2)-2a_2\beta_2(\gamma_1+1)c_2+2\alpha_2\beta_1c_2^2)+2a_2\beta_1\beta_2c_2^2(\gamma_1+1)\\
&\quad +\alpha_2\beta_1c_2(3\beta_1\gamma_1-\beta_2(\gamma_1+1)+\beta_1)+\alpha_2\beta_1^2c_1^3-3\alpha_2\beta_1^2c_2c_1^2)+\beta_1b_1^2\\
&\quad (\alpha_2\beta_2(\beta_2\gamma_1+\beta_1+\beta_2)+c_1(a_2\beta_2((\beta_1+\beta_2)\gamma_1-\beta_1+\beta_2)-2a_2\beta_1\beta_2c_2^2\\
&\quad +\alpha_2\beta_1(\beta_1+\beta_2)c_2)+a_2\beta_2c_2(-3\beta_1\gamma_1+\beta_2\gamma_1-\beta_1+\beta_2)-a_2\beta_1\beta_2c_1^3\\
&\quad +3a_2\beta_1\beta_2c_2c_1^2-2\alpha_2\beta_1^2c_2^2)-\beta_1^2\beta_2b_1(a_2(\beta_2+\beta_1((c_1-2c_2)c_2-1)+\beta_2c_1c_2)\\
&\quad +\alpha_2(\beta_1c_2+(\beta_2-\beta_1)c_1))+\beta_1^3\beta_2^2(a_2(c_2-c_1)+\alpha_2)\\
&\quad +\alpha_2b_1^5(-(c_1-c_2))(\gamma_1+1)^2]/[\beta_2(b_1^2+\beta_2^2)(-b_1\beta_1c_1+b_1^2(\gamma_1+1)+\beta_1^2)]\},\\
v_2 &= \{b_1=0,\ \beta_1=0\},\\
v_3 &= \{b_1=0,\ \beta_2=0\},\\
v_4 &= \{a_2=\alpha_2b_1/\beta_2,\ c_1=b_1(\gamma_1+1)/\beta_1+\beta_1/b_1\},\\
v_5 &= \{\beta_1=0,\ \gamma_1=-1\},\\
v_6 &= \{c_1=2c_2-b_1(\gamma_1+1)/\beta_1,\ \beta_2=0\},\\
v_7 &= \{c_1=(b_1(b_1\gamma_1+b_1+\beta_1c_2)-[b_1(-2\beta_1b_1^2c_2(\gamma_1+1)+\beta_1^2b_1(c_2^2+4\gamma_1)\\
&\quad +b_1^3(\gamma_1+1)^2-4\beta_1^3c_2)]^{1/2})/(2b_1\beta_1),\ \beta_2=0\},\\
v_8 &= \{c_1=(b_1(b_1\gamma_1+b_1+\beta_1c_2)+[b_1(-2\beta_1b_1^2c_2(\gamma_1+1)+\beta_1^2b_1(c_2^2+4\gamma_1)\\
&\quad +b_1^3(\gamma_1+1)^2-4\beta_1^3c_2)]^{1/2})/(2b_1\beta_1),\ \beta_2=0\},\\
v_9 &= \{\alpha_2=0,\ \beta_2=0\},\\
v_{10} &= \{c_1=(2b_1c_2+\beta_1-\beta_2)/(2b_1),\ \gamma_1=-(\beta_1(-2b_1c_2+\beta_1+\beta_2))/(2b_1^2)-1\},\\
v_{11} &= \{c_1=c_2,\ \beta_1=0,\ \beta_2=0\},\\
v_{12} &= \{\alpha_2=0,\ \beta_1=0,\ \beta_2=0\} \quad \text{and}\\
v_{13} &= \{\beta_1=0,\ \beta_2=0,\ \gamma_1=-1\}.
\end{aligned}
\tag{21}
$$

{v5}

The allowed real solutions which do not contradict (11) are v_1, v_7 and v_8 of (21). Then we must discuss these three cases.

Subcase 1.1.1: Consider the set v_1 of (21). We solve the third equation of (17) to obtain one of the following sets of real solutions

$$
\begin{aligned}
w_1 &= \{b_1=0\},\\
w_2 &= \{\beta_2=\alpha_2b_1/a_2\},\\
w_3 &= \{\beta_2=-b_1(b_1\gamma_1+b_1+\beta_1c_1-2\beta_1c_2)/\beta_1\},\\
w_4 &= \{\beta_2=-[(\beta_1b_1(c_1(c_2-c_1)+\gamma_1)+b_1^2(c_1-c_2)(\gamma_1+1)-\beta_1^2c_2)(\beta_1b_1(-2c_2\gamma_1\\
&\quad -c_1((c_1-c_2)^2+2))+b_1^2(\gamma_1+1)((c_1-c_2)^2+\gamma_1+1)+\beta_1^2(c_2^2+1))]\\
&\quad /[(\beta_1c_1-b_1\gamma_1)(-2b_1\beta_1c_1(\gamma_1+1)+b_1^2(\gamma_1+1)^2+\beta_1^2(c_1^2+1))]\},\\
u_5 &= \{a_2=0,\ b_1=0\},\\
w_6 &= \{a_2=0,\ \alpha_2=0\},\\
w_7 &= \{b_1=0,\ \beta_1=0\},\\
w_8 &= \{c_2=c_1,\ \gamma_1=\beta_1c_1/b_1\},\\
w_9 &= \{\beta_1=0,\ \gamma_1=-1\},\\
w_{10} &= \{b_1=0,\ c_1=0,\ c_2=0\},\\
w_{11} &= \{c_2=c_1,\ \beta_1=0,\ \gamma_1=-1\} \quad \text{and}\\
w_{12} &= \{c_2=c_1,\ \beta_1=0,\ \gamma_1=0\}.
\end{aligned}
\tag{22}
$$

{w5}

The allowed real solutions which do not contradict (11) are w_4 and w_8 of (22). Then we must consider these two cases.

Subcase 1.1.1.1: Consider the set w_4 of (22). Then we have now a continuous piecewise differential system formed by two systems (8). From (16) we get $(y_1-y_2)^2/4=0$ then we have no periodic orbits formed by (8) and (8) in this subcase.

Subcase 1.1.1.2: Consider the set w_8 of (22). Then we have now a continuous piecewise differential system formed by two systems (8). Solving the algebraic system (12) becomes

$$\frac{b_1^2\left(y_1-y_2\right)\left(b_1^2+\beta_1^2\right)\left(2\beta_1 b_1 c_1 y_1+2\beta_1 b_1 c_1 y_2+2b_1 c_1+2\beta_1 b_1^2 y_1 y_2+b_1^2 y_1+b_1^2 y_2+2\beta_1 c_1^2\right)}{\left(\beta_1 b_1 y_1+b_1+\beta_1 c_1\right)^2\left(\beta_1 b_1 y_2+b_1+\beta_1 c_1\right)^2}=0,$$

and

$$\frac{b_1^2\left(y_1-y_2\right)\left(b_1^2+\beta_2^2\right)\left(2\beta_2 b_1 c_1 y_1+2\beta_2 b_1 c_1 y_2+2b_1 c_1+2\beta_2 b_1^2 y_1 y_2+b_1^2 y_1+b_1^2 y_2+2\beta_2 c_1^2\right)}{\left(\beta_2 b_1 y_1+b_1+\beta_2 c_1\right)^2\left(\beta_2 b_1 y_2+b_1+\beta_2 c_1\right)^2}=0.$$

From these last two equation we get $y_1=y_2$ or $y_1=y_2=-c_1/b_1$, then we have no periodic orbits in this subcase.

Subcase 1.1.2: Consider the set v_7 of (21). In this case (14) and (15) give $y_1=-c_2/b_1$, and $y_2=-c_2/b_1$. Then we have no periodic orbits in this subcase.

Subcase 1.1.3: Consider the set v_8 of (21). Also in this case from (14) and (15) we get $y_1=-c_2/b_1$, and $y_2=-c_2/b_1$. Hence we have no periodic orbits in this subcase.

Subcase 1.2: Consider the set u_3 of (20). The first equation of (17) becomes

$$\beta_2\left(\alpha_2\beta_1-\alpha_1\beta_2\right)^2+2a_2\beta_2\beta_1 c_2\left(\alpha_2\beta_1-\alpha_1\beta_2\right)+a_2^2\beta_1^2\left(-\beta_2\gamma_1+\beta_1\gamma_2+\left(\beta_2-\beta_1\right)c_2^2\right)=0,$$

which gives one of the following sets of solutions

$$\begin{aligned}
s_1 &= \{\gamma_2=(-\beta_2(\alpha_2\beta_1-\alpha_1\beta_2)^2+2a_2\beta_2\beta_1c_2(\alpha_1\beta_2-\alpha_2\beta_1)+a_2^2\beta_1^2(\beta_2\gamma_1+(\beta_1-\beta_2)c_2^2))\\
&\quad /(a_2^2\beta_1^3)\},\\
s_2 &= \{a_2=0,\ \alpha_1=\alpha_2\beta_1/\beta_2\},\\
s_3 &= \{a_2=0,\ \beta_2=0\},\\
s_4 &= \{\alpha_1=0,\ \beta_1=0\}\quad \text{and}\\
s_5 &= \{\beta_1=0,\ \beta_2=0\}.
\end{aligned}$$

The allowed solution is s_1 which gives

$$\gamma_2=\frac{-\beta_2\left(\alpha_2\beta_1-\alpha_1\beta_2\right)^2+2a_2\beta_2\beta_1c_2\left(\alpha_1\beta_2-\alpha_2\beta_1\right)+a_2^2\beta_1^2\left(\beta_2\gamma_1+\left(\beta_1-\beta_2\right)c_2^2\right)}{a_2^2\beta_1^3}.$$

Then (14) and (15) become

$$y_1=\frac{\left(\alpha_2\beta_1-\alpha_1\beta_2\right)^2+2a_2\beta_1c_2\left(\alpha_2\beta_1-\alpha_1\beta_2\right)+a_2^2\beta_1^2\left(c_2^2-\gamma_1\right)}{a_2^2\beta_1^3},$$

and

$$y_2=\frac{\left(\alpha_2\beta_1-\alpha_1\beta_2\right)^2+2a_2\beta_1c_2\left(\alpha_2\beta_1-\alpha_1\beta_2\right)+a_2^2\beta_1^2\left(c_2^2-\gamma_1\right)}{a_2^2\beta_1^3}.$$

So $y_1=y_2$ which gives no periodic orbits in this subcase.

Subcase 1.3: Consider the set u_6 of (20). The first of (17) becomes

$$\begin{aligned}
&-\beta_1(\alpha_2b_1-a_2\beta_2)(b_1^3(c_1(\beta_1\gamma_2-\beta_2\gamma_2+\beta_1+\beta_2+4\beta_2c_2^2)-c_2(\beta_1\gamma_2-\beta_2\gamma_2+\beta_1+2\beta_2(c_2^2+1))\\
&-2\beta_2c_2c_1^2)+\beta_2b_1^2(\beta_2(2c_1^2-3c_2c_1+c_2^2-\gamma_2+1)+\beta_1(c_2^2-c_1c_2+2\gamma_2+1))+\beta_2^2b_1((\beta_1+\beta_2)c_1\\
&-3\beta_1c_2)+b_1^4(2(c_1-c_2)^2+1)(\gamma_2+1)+\beta_1\beta_2^3)=0,
\end{aligned}$$

which gives one of the following sets of real solutions

$$\begin{aligned}
s_1 &= \{\beta_1=0\},\\
s_2 &= \{\beta_1=[b_1(\beta_2b_1^2(c_1(-4c_2^2+\gamma_2-1)+c_2(2c_2^2-\gamma_2+2)+2c_2c_1^2)+\beta_2^2b_1(-2c_1^2+3c_2c_1\\
&\quad -c_2^2+\gamma_2-1)+b_1^3(-(2(c_1-c_2)^2+1))(\gamma_2+1)-\beta_2^3c_1)]/[\beta_2b_1^2(c_2^2-c_1c_2+2\gamma_2+1)\\
&\quad +\beta_2^2b_1(c_1-3c_2)+b_1^3(c_1-c_2)(\gamma_2+1)+\beta_2^3]\},\\
s_3 &= \{\beta_2=\alpha_2b_1/a_2\},\\
s_4 &= \{a_2=0,\ b_1=0\},\\
s_5 &= \{a_2=0,\ \alpha_2=0\},\\
s_6 &= \{b_1=0,\ \beta_2=0\}\quad \text{and}\\
s_7 &= \{\beta_2=0,\ \gamma_2=-1\}.
\end{aligned}$$

The allowed solution is s_2 which gives

$$\beta_1=[b_1(\beta_2 b_1^2(c_1(-4c_2^2+\gamma_2-1)+c_2(2c_2^2-\gamma_2+2)+2c_2c_1^2)+\beta_2^2 b_1(-2c_1^2+3c_2c_1 -c_2^2+\gamma_2-1)+b_1^3(-(2(c_1-c_2)^2+1))(\gamma_2+1)-\beta_2^3c_1)]/[\beta_2 b_1^2(c_2^2-c_1c_2+2\gamma_2+1) +\beta_2^2 b_1(c_1-3c_2)+b_1^3(c_1-c_2)(\gamma_2+1)+\beta_2^3].$$

Then (14) and (15) become

$$y_1=\frac{\beta_2\left(c_2^2-\gamma_2\right)-b_1c_2\left(\gamma_2+1\right)}{-b_1\beta_2c_2+b_1^2\left(\gamma_2+1\right)+\beta_2^2},\quad \text{and}\quad y_2=\frac{\beta_2\left(c_2^2-\gamma_2\right)-b_1c_2\left(\gamma_2+1\right)}{-b_1\beta_2c_2+b_1^2\left(\gamma_2+1\right)+\beta_2^2}.$$

Therefore $y_1=y_2$ which gives no periodic orbits in this subcase.

Subcase 1.4: Consider the set u_8 of (20). Solving the first and the third equation of (17) simultaneously we get one of the following sets of real solutions

$$\begin{array}{rcl}
s_1 &=& \{\alpha_2=0\},\\
s_2 &=& \{\beta_1=0\},\\
s_3 &=& \{b_1=0,\ \beta_2=0\},\\
s_4 &=& \{\beta_1=[b_1^2(\beta_2 b_1(-((c_1-c_2)^2+2))(c_1-c_2)-b_1^2(2(c_1-c_2)^4+3(c_1-c_2)^2+1)-\beta_2^2)]\\
& & /[(b_1(c_1-c_2)+\beta_2)(2\beta_2b_1(c_1-c_2)+b_1^2((c_1-c_2)^2+1)+\beta_2^2)],\ \gamma_2=\beta_2c_1/b_1+(c_1-c_2)^2\},\\
s_5 &=& \{\beta_1=2b_1(c_1-c_2)+\beta_2,\ \gamma_2=-[-\beta_2b_1c_2+b_1^2+\beta_2^2]/b_1^2\},\\
s_6 &=& \{b_1=0,\ c_1=0,\ \beta_1=0\},\\
s_7 &=& \{b_1=0,\ \beta_1=0,\ \beta_2=0\}\quad \text{and}\\
s_8 &=& \{\beta_2=0,\ \gamma_2=-1\}.
\end{array}$$

The allowed solution is s_4 which gives

$$\beta_1=\frac{b_1^2\left(\beta_2b_1\left(-\left((c_1-c_2)^2+2\right)\right)(c_1-c_2)-b_1^2\left(2\left(c_1-c_2\right)^4+3\left(c_1-c_2\right)^2+1\right)-\beta_2^2\right)}{\left(b_1\left(c_1-c_2\right)+\beta_2\right)\left(2\beta_2b_1\left(c_1-c_2\right)+b_1^2\left(\left(c_1-c_2\right)^2+1\right)+\beta_2^2\right)},$$

and

$$\gamma_2=\frac{\beta_2c_1}{b_1}+\left(c_1-c_2\right)^2.$$

Then (14) and (15) become

$$y_1=\frac{\beta_2b_1c_1\left(c_2-c_1\right)-b_1^2c_2\left(c_1^2-2c_2c_1+c_2^2+1\right)-\beta_2^2c_1}{b_1\left(\beta_2b_1\left(c_1-c_2\right)+b_1^2\left(c_1^2-2c_2c_1+c_2^2+1\right)+\beta_2^2\right)},$$

and

$$y_2=\frac{\beta_2b_1c_1\left(c_2-c_1\right)-b_1^2c_2\left(c_1^2-2c_2c_1+c_2^2+1\right)-\beta_2^2c_1}{b_1\left(\beta_2b_1\left(c_1-c_2\right)+b_1^2\left(c_1^2-2c_2c_1+c_2^2+1\right)+\beta_2^2\right)}.$$

Hence $y_1=y_2$ which gives no periodic orbits in this subcase.

Case 2: Assume that $\beta_1\left(-b_1\beta_2c_2+b_1^2\left(\gamma_2+1\right)+\beta_2^2\right)=0$. This case is divided in two subcases regarding d_1 and d_2 of (19).

Subcase 2.1: Consider the set d_1 of (19). Solving the first and the second equation of (17) simultaneously we get one of the following sets of real solutions

$$\begin{array}{rcl} w_1 &=& \{b_1=0\},\\ w_2 &=& \{b_1=0,\ \beta_2=0\},\\ w_3 &=& \{c_1=-[a_2\beta_2^2b_1(\gamma_1+1)+\beta_2b_1^2(a_2c_2(\gamma_1+1)-\alpha_2(\gamma_1+1)+\alpha_1)+\alpha_1\beta_2^3\\ && +\alpha_2b_1^3(-c_2)(\gamma_1+1)]/[b_1^2(\gamma_1+1)(\alpha_2b_1-a_2\beta_2)],\\ && \gamma_2=[-\beta_2b_1(a_2\gamma_1+a_2-\alpha_1c_2)-\alpha_1\beta_2^2+b_1^2(\alpha_2\gamma_1-\alpha_1+\alpha_2)]/[\alpha_1b_1^2]\},\\ w_4 &=& \{\alpha_1=0,\ \alpha_2=\frac{a_2\beta_2}{b_1}\},\\ u_5 &=& \{\alpha_1=0,\ \gamma_1=-1\},\\ w_6 &=& \{a_2=0,\ b_1=0,\ \alpha_1=0\},\\ w_7 &=& \{b_1=0,\ c_1=0,\ \alpha_1=0\},\\ w_8 &=& \{b_1=0,\ \alpha_1=0,\ \gamma_1=-1\},\\ w_9 &=& \{\alpha_1=0,\ \alpha_2=0,\ \beta_2=0\},\\ w_{10} &=& \{\alpha_1=0,\ \beta_2=0,\ \gamma_1=-1\},\\ w_{11} &=& \{\alpha_2=0,\ \beta_2=0,\ \gamma_2=-1\},\\ w_{12} &=& \{\beta_2=0,\ \gamma_1=-1,\ \gamma_2=-1\} \quad \text{and}\\ w_{13} &=& \{a_2=0,\ \alpha_2=0,\ \beta_2=0,\ \gamma_2=-1\}. \end{array} \tag{23}$$ {w6}

The allowed real solutions which do not contradict (11) are w_3 and w_{12} of (23). Then we must discuss these two subcases.

Subcase 2.1.1: Consider the set w_3 of (23). Then (14) and (15) become

$$y_1=\frac{a_2\beta_2^2b_1\left(\gamma_1+1\right)+\beta_2b_1^2\left(a_2c_2\left(\gamma_1+1\right)-\alpha_2\left(\gamma_1+1\right)+\alpha_1\right)+\alpha_1\beta_2^3+\alpha_2b_1^3\left(-c_2\right)\left(\gamma_1+1\right)}{b_1^3\left(\gamma_1+1\right)\left(\alpha_2b_1-a_2\beta_2\right)},$$

and

$$y_2=\frac{a_2\beta_2^2b_1\left(\gamma_1+1\right)+\beta_2b_1^2\left(a_2c_2\left(\gamma_1+1\right)-\alpha_2\left(\gamma_1+1\right)+\alpha_1\right)+\alpha_1\beta_2^3+\alpha_2b_1^3\left(-c_2\right)\left(\gamma_1+1\right)}{b_1^3\left(\gamma_1+1\right)\left(\alpha_2b_1-a_2\beta_2\right)}.$$

So $y_1=y_2$ which gives no periodic orbits in this subcase.

Subcase 2.1.2: Consider the set w_{12} of (23). First integrals have a singularity on the $y-$axis. Then we have no periodic orbits in this subcase.

Subcase 2.2: Consider the set d_2 of (19). Then we have

$$\gamma_2=\frac{\beta_2b_1c_2-b_1^2-\beta_2^2}{b_1^2}.$$

Solving the first and the second equation of (17) simultaneously we get one of the following sets of real solutions

$$\begin{array}{rcl} w_1 &=& \{b_1=0,\ \beta_1=0\},\\ w_2 &=& \{b_1=0,\ \beta_2=0\},\\ w_3 &=& \{\alpha_1=\frac{a_1\beta_1}{b_1},\ \alpha_2=\frac{a_2\beta_2}{b_1}\},\\ w_4 &=& \{\alpha_2=0,\ \beta_2=0\},\\ u_5 &=& \{\alpha_1=\frac{\beta_1((a_1-a_2)\beta_2b_1^2+a_1\beta_2^3-a_2\beta_1^2\beta_2+\alpha_2\beta_1^2b_1+\alpha_2b_1^3)}{b_1\beta_2(b_1^2+\beta_2^2)},\ \gamma_1=-\frac{-\beta_1b_1c_1+b_1^2+\beta_1^2}{b_1^2}\},\\ w_6 &=& \{a_1=0,\ a_2=0,\ b_1=0\},\\ w_7 &=& \{a_2=0,\ b_1=0,\ \beta_1=0\},\\ w_8 &=& \{b_1=0,\ \beta_1=0,\ \beta_2=0\},\\ w_9 &=& \{\beta_1=0,\ \beta_2=0,\ \gamma_1=-1\} \quad \text{and}\\ w_{10} &=& \{\alpha_2=0,\ \beta_2=0,\ \gamma_1=-\frac{-\beta_1b_1c_1+b_1^2+\beta_1^2}{b_1^2}\}. \end{array} \tag{24}$$ {w7}

The allowed real solutions which do not contradict (11) are w_5 and w_9 of (24). Then we must discuss these two cases.

Subcase 2.2.1: Consider the set w_5 of (24). Solving the third and the fourth equation of (17) simultaneously we get one of the following sets of real solutions

$$\begin{array}{rcl} s_1 &=& \{\beta_1=0\}, \\ s_2 &=& \{\beta_2=\alpha_2 b_1/a_2\}, \\ s_3 &=& \{a_2=0,\ b_1=0\}, \\ s_4 &=& \{a_2=0,\ \alpha_2=0\}, \\ s_5 &=& \{a_1=\frac{\beta_2 b_1^2(a_2(3(c_1-c_2)^2+1)+3\alpha_2(c_2-c_1))+3a_2\beta_2^2 b_1(c_1-c_2)+a_2\beta_2^3-3\alpha_2 b_1^3(c_1-c_2)^2}{\beta_2(b_1^2+\beta_2^2)},\ \beta_1=2b_1(c_1-c_2)+\beta_2\}, \\ s_6 &=& \{b_1=0,\ \beta_1=0,\ \beta_2=0\}, \\ s_7 &=& \{c_2=c_1,\ \beta_1=0,\ \beta_2=0\} \quad \text{and} \\ s_8 &=& \{\alpha_2=0,\ \beta_1=2b_1(c_1-c_2),\ \beta_2=0\}. \end{array}$$

The only allowed real solution which does not contradict (11) is s_5. Then we have now a continuous piecewise differential systems formed by two systems (8). Now we will solve the algebraic system (12) which equivalent to

$$\frac{(y_1-y_2)\left(-2b_1c_1+4b_1c_2+b_1^2y_1+b_1^2y_2-2\beta_2\right)}{(2b_1c_2-b_1c_1+b_1^2y_1-\beta_2)^2\,(2b_1c_2-b_1c_1+b_1^2y_2-\beta_2)^2}=0,$$

and

$$\frac{(y_1-y_2)\left(2b_1c_2+b_1^2y_1+b_1^2y_2-2\beta_2\right)}{(b_1c_2+b_1^2y_1-\beta_2)^2\,(b_1c_2+b_1^2y_2-\beta_2)^2}=0.$$

From these last two equation we get $y_1=y_2$, then we have no periodic orbits in this subcase.

Subcase 2.2.2: Consider the set w_9 of (24). Then the third and the fourth equation of (17) become

$$\alpha_1\alpha_2 b_1\,(c_1-c_2)\,(c_1+c_2)=0, \quad \text{and} \quad 2\alpha_1\alpha_2 b_1^2\,(c_1-c_2)=0.$$

The only solution of these last equations which do not contradict (11) is $c_2=c_1$. In this case first integrals have a singularity on the $y-$axis, then no periodic orbits in this subcase. ■

Proof of statement (f) *of Theorem* 1. Consider the generalized isochronous quadratic systems (8) and (9). In order that the piecewise differential systems formed by systems (8) and (9) be continuous, they must coincide on $x=0$, which means that the coefficients of the system must satisfy the following algebraic system

$$\begin{aligned} &-3a_2b_1\beta_2\gamma_1c_1-16a_1b_2\beta_1c_2\gamma_2-3a_2b_1\beta_2c_1+3a_1b_2\beta_1c_2-3a_2\beta_1\beta_2\gamma_1+3a_1\beta_1\beta_2\gamma_2\\ &+3a_2\beta_1\beta_2c_1^2+4a_1\beta_1\beta_2c_2^2+3\alpha_2b_2\beta_1\gamma_1-3\alpha_1b_1\beta_2\gamma_2-3\alpha_2b_2\beta_1c_1^2-4\alpha_1b_1\beta_2c_2^2\\ &+3\alpha_2b_1b_2\gamma_1c_1+16\alpha_1b_1b_2c_2\gamma_2+3\alpha_2b_1b_2c_1-3\alpha_1b_1b_2c_2=0,\\[4pt] &-3a_2\beta_2b_1^2\gamma_1-16a_1b_2^2\beta_1\gamma_2-3a_2\beta_2b_1^2+3a_1b_2^2\beta_1+3a_2\beta_1\beta_2b_1c_1-8a_1b_2\beta_1\beta_2c_2+3a_1\beta_1\beta_2^2\\ &-3a_2\beta_1^2\beta_2-3\alpha_1\beta_2^2b_1+3\alpha_2b_2\beta_1^2+3\alpha_2b_2b_1^2\gamma_1+16\alpha_1b_2^2b_1\gamma_2+3\alpha_2b_2b_1^2-3\alpha_1b_2^2b_1\\ &-3\alpha_2b_2\beta_1b_1c_1+8\alpha_1b_2\beta_2b_1c_2=0,\\[4pt] &4b_2^2\beta_2=0,\\[4pt] &3a_2\alpha_1\beta_2\gamma_1-3a_1\alpha_2\beta_1\gamma_2-3a_1\alpha_2b_2\gamma_1c_1-16a_2\alpha_1b_1c_2\gamma_2-3a_1\alpha_2b_2c_1+3a_2\alpha_1b_1c_2\\ &-3a_2\alpha_1\beta_2c_1^2-4a_1\alpha_2\beta_1c_2^2+3a_1a_2\beta_2\gamma_1c_1+16a_1a_2\beta_1c_2\gamma_2+3a_1a_2\beta_2c_1-3a_1a_2\beta_1c_2\\ &-3\alpha_1\alpha_2b_2\gamma_1+3\alpha_1\alpha_2b_1\gamma_2+3\alpha_1\alpha_2b_2c_1^2+4\alpha_1\alpha_2b_1c_2^2=0,\\[4pt] &3a_2\alpha_1\beta_1\beta_2-3a_1\alpha_2\beta_1\beta_2-16a_2\alpha_1b_1b_2\gamma_2-3a_1\alpha_2b_1b_2\gamma_1+3a_2\alpha_1b_1b_2-3a_1\alpha_2b_1b_2\\ &+3a_1a_2b_1\beta_2\gamma_1+16a_1a_2b_2\beta_1\gamma_2-3a_1a_2b_2\beta_1+3a_1a_2b_1\beta_2-6a_2\alpha_1b_1\beta_2c_1-16a_2\alpha_1b_1\beta_2c_2\\ &-3a_1\alpha_2b_2\beta_1c_1-8a_1\alpha_2b_2\beta_1c_2+3a_1a_2\beta_1\beta_2c_1+16a_1a_2\beta_1\beta_2c_2-3\alpha_2\alpha_1b_2\beta_1+3\alpha_2\alpha_1b_1\beta_2\\ &+6\alpha_2\alpha_1b_1b_2c_1+8\alpha_2\alpha_1b_1b_2c_2=0,\\[4pt] &-16a_2\beta_2b_2-3a_2b_1\beta_2+4\alpha_2b_2^2+3\alpha_2b_1b_2=0, \end{aligned} \tag{25}$$

{cfpc.2.3}

together with the conditions (11).

Solving the third and the sixth equation of the algebraic system (25) we get one of the following sets of solutions

$$\begin{aligned} s_1 &= \{a_2=0,\ b_2=0\},\\ s_2 &= \{b_1=0,\ b_2=0\},\\ s_3 &= \{b_2=0,\ \beta_2=0\},\\ s_4 &= \{b_1=-4b_2/3,\ \beta_2=0\} \quad \text{and}\\ s_5 &= \{\alpha_2=0,\ \beta_2=0\}. \end{aligned} \tag{26}$$ {s7}

The allowed solutions which does not contradict (11) are s_2 and s_4 of (26). Then we have two cases.

Case 1: We consider s_2 of (26), then we have $b_1=0$ and $b_2=0$. Solving the second and the fifth equation of (25) we obtain one of the following sets of solutions

$$\begin{aligned} u_1 &= \{\beta_1=0\},\\ u_2 &= \{\beta_2=0\},\\ u_3 &= \{a_1=0,\ a_2=0\},\\ u_4 &= \{\alpha_2=a_2(\tfrac{\alpha_1}{a_1}+c_1+\tfrac{16c_2}{3}),\ \beta_2=\tfrac{a_2\beta_1}{a_1}\} \quad \text{and}\\ u_5 &= \{a_1=0,\ \alpha_1=0,\ \beta_1=0\}. \end{aligned}$$

The only allowed solution which doe not contradict (11) is u_4. Then we have

$$\alpha_2=a_2\left(\frac{\alpha_1}{a_1}+c_1+\frac{16c_2}{3}\right) \text{ and } \beta_2=\frac{a_2\beta_1}{a_1}.$$

Solving the first and the fourth equation of (25) we obtain one of the following sets of real solutions

$$\begin{aligned} z_1 &= \{a_2=0\},\\ z_2 &= \{\beta_1=0\},\\ z_3 &= \{a_2=\tfrac{a_1c_2(64c_2^2+9)}{9(c_1^3+c_1)},\ \gamma_2=-\tfrac{c_2(-(64c_2^2+9)\gamma_1+12c_2c_1^3+(64c_2^2+9)c_1^2+12c_2c_1)}{9(c_1^3+c_1)}\} \quad \text{and}\\ z_4 &= \{c_1=0,\ c_2=0,\ \gamma_2=\tfrac{a_2\gamma_1}{a_1}\}. \end{aligned} \tag{27}$$ {z}

The allowed solutions which do not contradict (11) are z_3 and s_4 of (27). Then we get two subcases.

Subcase 1.1: We consider z_3 of (27). We have now a piecewise continuous differential systems formed by systems (8) and (9). The algebraic system (12) is equivalent to the following equations

$$\begin{aligned} &\beta_1(y_1-y_2)(-2\gamma_1c_1^2+\beta_1c_1^2(-y_1)-\beta_1c_1^2y_2-2c_1^2+2\gamma_1^2+2\gamma_1+2\beta_1\gamma_1y_1+2\beta_1\gamma_1y_2+2\beta_1^2y_1y_2\\ &+\beta_1y_1+\beta_1y_2)/[(\gamma_1+\beta_1y_1+1)^2(\gamma_1+\beta_1y_2+1)^2]=0, \end{aligned}$$

and

$$\begin{aligned} &64\beta_1c_2^2(64c_2^2+9)(y_1-y_2)(-6\gamma_1c_1^3-32c_2\gamma_1c_1^2-6\gamma_1c_1+16c_2\gamma_1^2+16\beta_1c_2\gamma_1y_1+16\beta_1c_2\gamma_1y_2\\ &-3\beta_1c_1^3y_1-3\beta_1c_1^3y_2-16\beta_1c_2c_1^2y_1-16\beta_1c_2c_1^2y_2-3\beta_1c_1y_1-3\beta_1c_1y_2+16\beta_1^2c_2y_1y_2+6c_1^5\\ &+16c_2c_1^4+6c_1^3)/[c_1(c_1^2+1)(-16c_2\gamma_1-16\beta_1c_2y_1+3c_1^3+16c_2c_1^2+3c_1)(-16c_2\gamma_1-16\beta_1c_2y_2\\ &+3c_1^3+16c_2c_1^2+3c_1)]=0. \end{aligned}$$

Solving the first equation for y_1 we get

$$y_1=\frac{c_1^2\left(2\gamma_1+\beta_1y_2+2\right)-2\gamma_1\left(\gamma_1+1\right)-y_2\left(2\beta_1\gamma_1+\beta_1\right)}{\beta_1\left(-c_1^2+2\gamma_1+2\beta_1y_2+1\right)},$$

replacing this value into the second equation which, by excluding the trivial case $y_1=y_2$, becomes

$$\frac{2\left(c_1^2+1\right)\left(3c_1+8c_2\right)\left(c_1^2-\gamma_1-\beta_1y_2\right)^2}{c_1^2-2\gamma_1-2\beta_1y_2-1}=0.$$

Finally we obtain $y_1=y_2=(c_1^2-\gamma_1)/\beta_1$. Then we have no periodic orbits, and then no limit cycles formed by systems (8) and (9) in this subcase.

Subcase 1.2: We consider z_4 of (27). We have now a piecewise continuous differential systems formed by systems (8) and (9). The algebraic system (12) is equivalent to the following equations

$$\frac{\beta_1\left(y_1-y_2\right)\left(2\gamma_1^2+2\gamma_1+2\beta_1\gamma_1y_1+2\beta_1\gamma_1y_2+2\beta_1^2y_1y_2+\beta_1y_1+\beta_1y_2\right)}{\left(\gamma_1+\beta_1y_1+1\right)^2\left(\gamma_1+\beta_1y_2+1\right)^2},$$

and

$$\frac{(y_1-y_2)\left(-16a_2\gamma_1^2+6a_1\gamma_1-16a_2\beta_1\gamma_1y_1-16a_2\beta_1\gamma_1y_2-16a_2\beta_1^2y_1y_2+3a_1\beta_1y_1+3a_1\beta_1y_2\right)}{a_1\left(-16a_2\gamma_1-16a_2\beta_1y_1+3a_1\right)\left(-16a_2\gamma_1-16a_2\beta_1y_2+3a_1\right)}.$$

Solving the first equation for y_1 we get

$$y_1=-\frac{2\gamma_1\left(\gamma_1+1\right)+y_2\left(2\beta_1\gamma_1+\beta_1\right)}{\beta_1\left(2\gamma_1+2\beta_1y_2+1\right)},$$

replacing this value into the second equation which, by excluding the trivial case $y_1=y_2$, becomes

$$\frac{2\left(3a_1+8a_2\right)\left(\gamma_1+\beta_1y_2\right)^2}{2\gamma_1+2\beta_1y_2+1}=0.$$

Finally we obtain $y_1=y_2=-\gamma_1/\beta_1$. Then we have no periodic orbits, and then no limit cycles in this subcase.

Case 2: We consider s_4 of (26). Solving the second equation of (25) we obtain one of the following sets of solutions

$$\begin{array}{rcl} s_1 &=& \{b_2=0\},\\ s_2 &=& \{\gamma_1=-\frac{3\beta_1b_2(a_1(3-16\gamma_2)+4\alpha_2c_1)+9\alpha_2\beta_1^2+4b_2^2(\alpha_1(3-16\gamma_2)+4\alpha_2)}{16\alpha_2b_2^2}\},\\ s_3 &=& \{b_2=0,\ \alpha_2=0\},\\ s_4 &=& \{b_2=0,\ \beta_1=0\},\\ s_5 &=& \{\alpha_1=-\frac{3a_1\beta_1}{4b_2},\ \alpha_2=0\}\quad\text{and}\\ s_6 &=& \{\alpha_2=0,\ \gamma_2=\frac{3}{16}\}. \end{array}$$

The allowed solution is s_2, which means that

$$\gamma_1=-\frac{3\beta_1b_2\left(a_1\left(3-16\gamma_2\right)+4\alpha_2c_1\right)+9\alpha_2\beta_1^2+4b_2^2\left(\alpha_1\left(3-16\gamma_2\right)+4\alpha_2\right)}{16\alpha_2b_2^2}.$$

Solving the first and the fifth equation of (25) we obtain one of the following sets of real solutions

$$\begin{array}{rcl} z_1 &=& \{\alpha_1=-\frac{3a_1\beta_1}{4b_2},\alpha_2=0\},\\ z_2 &=& \{a_2=-[b_2(12b_2\beta_1(a_1(18c_1^2+48c_2c_1+32c_2^2+9)-3\alpha_1(3c_1+4c_2))\\ && -27\beta_1^2(3a_1c_1+4a_1c_2+3\alpha_1)+32\alpha_1b_2^2(3c_1+4c_2)^2)]/[9(16b_2^2\beta_1+9\beta_1^3)],\\ && \alpha_2=-\frac{b_2(16\gamma_2-3)(4b_2(3c_1+4c_2)-9\beta_1)(3a_1\beta_1+4\alpha_1b_2)}{9(16b_2^2\beta_1+9\beta_1^3)}\},\\ z_3 &=& \{\alpha_1=0,\ \beta_1=0\},\\ z_4 &=& \{\alpha_2=0,\ \gamma_2=\frac{3}{16}\},\\ z_5 &=& \{a_1=0,\ \alpha_1=0,\ \alpha_2=0\},\\ z_6 &=& \{a_2=-\frac{3a_1}{4},\ c_2=-\frac{3c_1}{4},\ \beta_1=0\},\\ z_7 &=& \{a_2=\frac{a_1(9-48\gamma_2)+8\alpha_2(3c_1+4c_2)}{64\gamma_2-12},\ \alpha_1=0,\ \beta_1=0\},\\ z_8 &=& \{c_2=-\frac{3c_1}{4},\ \beta_1=0,\ \gamma_2=\frac{3}{16}\},\\ z_9 &=& \{\alpha_2=0,\ \beta_1=0,\ \gamma_2=\frac{3}{16}\}\quad\text{and}\\ z_{10} &=& \{a_1=0,\ a_2=\frac{2\alpha_2(3c_1+4c_2)}{16\gamma_2-3},\ \alpha_1=0,\ \beta_1=0\}. \end{array} \tag{28}$$ {z1}

The allowed solutions which do not contradict (11) are z_2, z_6 and z_8 of (28). Then we must discuss these three subcases.

Subcase 2.1: We consider z_2 of (28). We solve now the fourth equation of (25) to obtain one of the following sets of real solutions

$$\begin{array}{rcl} v_1 &=& \{b_2=0\},\\ v_2 &=& \{\beta_1=-\frac{4\alpha_1b_2}{3a_1}\},\\ v_3 &=& \{\beta_1=-\frac{b_2(3c_1+4c_2)(9c_1^2+24c_2c_1+16c_2^2-12\gamma_2+9)}{27\gamma_2}\},\\ v_4 &=& \{\gamma_2=\frac{3}{16}\},\\ v_5 &=& \{a_1=0,\ b_2=0\},\\ v_6 &=& \{a_1=0,\ \alpha_1=0\},\\ v_7 &=& \{b_2=0,\ \gamma_2=0\}\quad\text{and}\\ v_8 &=& \{c_2=-\frac{3c_1}{4},\ \gamma_2=0\}. \end{array} \tag{29}$$ {z2}

The allowed solutions which do not contradict (11) are v_3 and v_8 of (29). Then we must discuss these two subcases.

Subcase 2.1.1: We consider v_3 of (29). We have now a piecewise continuous differential systems formed by systems (8) and (9). We solve the algebraic system (12) we get two different pairs (y_1, y_2). The first pair is

$$y_{1\,1} = -\frac{\sqrt{3}}{8}\sqrt{\mathcal{R}_1} - \frac{\sqrt{3}}{8}\sqrt{\mathcal{R}_2} + \frac{\frac{6\gamma_2}{3c_1+4c_2} - 8c_2}{8b_2},$$

and

$$y_{2\,1} = \frac{\sqrt{3}}{8}\sqrt{\mathcal{R}_1} - \frac{\sqrt{3}}{8}\sqrt{\mathcal{R}_2} + \frac{\frac{6\gamma_2}{3c_1+4c_2} - 8c_2}{8b_2}.$$

Where

$$\mathcal{R}_1 = \frac{4\sqrt{3}b_2\left(3c_1+4c_2\right)\gamma_2\sqrt{\mathcal{R}_2} + 72\gamma_2 c_1^2 + 192c_2\gamma_2 c_1 + 128c_2^2\gamma_2 - 27c_1^2 - 72c_2c_1 - 48c_2^2 - 24\gamma_2^2}{b_2^2\left(3c_1+4c_2\right)^2},$$

and

$$\mathcal{R}_2 = \frac{4\gamma_2\left(\frac{3\gamma_2}{(3c_1+4c_2)^2} + 2\right) - 3}{b_2^2}.$$

The second pair is

$$y_{1\,2} = -\frac{\sqrt{3}}{8}\sqrt{\mathcal{S}} + \frac{\sqrt{3}}{8}\sqrt{\mathcal{R}_2} + \frac{\frac{6\gamma_2}{3c_1+4c_2} - 8c_2}{8b_2},$$

and

$$y_{2\,2} = \frac{\sqrt{3}}{8}\sqrt{\mathcal{S}} + \frac{\sqrt{3}}{8}\sqrt{\mathcal{R}_2} + \frac{\frac{6\gamma_2}{3c_1+4c_2} - 8c_2}{8b_2}.$$

Where

$$\mathcal{S} = \frac{-4\sqrt{3}b_2\left(3c_1+4c_2\right)\gamma_2\sqrt{\mathcal{R}_2} + 72\gamma_2 c_1^2 + 192c_2\gamma_2 c_1 + 128c_2^2\gamma_2 - 27c_1^2 - 72c_2c_1 - 48c_2^2 - 24\gamma_2^2}{b_2^2\left(3c_1+4c_2\right)^2}.$$

Then in this subcase we can have at most two limit cycles formed by systems (8) and (9).

We have $y_{2\,2} > y_{1\,2} > y_{2\,1} > y_{1\,1}$ and since the invariant straight line

$$\begin{aligned} y = \;& \frac{3\left(3c_1+4c_2\right)\left(3c_1+8c_2\right)\gamma_2 - 36\gamma_2^2 + 27\gamma_2 - c_2\left(3c_1+4c_2\right)\left(\left(3c_1+4c_2\right)^2+9\right)}{b_2\left(3c_1+4c_2\right)\left(\left(3c_1+4c_2\right)^2 - 12\gamma_2 + 9\right)} \\ & + \frac{27\alpha_1\gamma_2}{b_2\left(3c_1+4c_2\right)\left(\left(3c_1+4c_2\right)^2 - 12\gamma_2 + 9\right)}x. \end{aligned}$$

intersects the $y-$axis at

$$y_0 = -\frac{4c_2^2 + 3c_1c_2 - 3\gamma_2}{3b_2c_1 + 4b_2c_2},$$

and because $y_0 \in [y_{1\,2}, y_{2\,2}]$ or $y_0 \in [y_{1\,1}, y_{2\,1}]$ $(y_0 \notin (-\infty, y_{1\,1}) \cup (y_{2\,1}, y_{1\,2}) \cup (y_{2\,2}, +\infty))$ we conclude that we have at most one limit cycle.

Then the unique limit cycle, if it exists, must be constructed in the segment $[y_{1\,2}, y_{2\,2}]$ or on the other segment $[y_{1\,1}, y_{2\,1}]$. In what follows we prove that there is no limit cycle under these conditions, for this reason, we consider the generalized system (10), and from all the sets of parameters that give continuity we choose only $\beta_2 = 0$, then the generalized system (10) becomes

$$\begin{aligned} \dot{x} &= -\frac{1}{3\alpha_2}(16\gamma_2 + 16\alpha_2 x - 3)(a_2x + b_2y + c_2), \\ \dot{y} &= \frac{1}{3\alpha_2 b_2}\left(a_2(b_2y+c_2)(16\gamma_2 + 8\alpha_2 x - 3) + a_2^2 x(16\gamma_2 + 12\alpha_2 x - 3) - 4\alpha_2(b_2y+c_2)^2 - 3\alpha_2(\gamma_2 + \alpha_2 x)\right), \end{aligned}$$

which is equivalent to the following ordinary differential equation

$$\frac{dy}{dx} = \frac{\dfrac{\dfrac{3\alpha_2\left(\gamma_2+\alpha_2x\right)}{a_2x+b_2y(x)+c_2} + a_2\left(-16\gamma_2 - 12\alpha_2x + 3\right) + 4\alpha_2c_2}{b_2} + 4\alpha_2 y(x)}{16\gamma_2 + 16\alpha_2 x - 3}. \qquad (30) \quad \{odeGnle4\}$$

Solving the initial value problem (30) and $y(0)=y_0$, we obtain the following set of solution

$$y(x)=-\frac{b_2\sqrt{W_1}\sqrt{\frac{1}{b_2^2{W_1}^{3/2}}}\sqrt{W_2}+8a_2x+8c_2}{8\,b_2}\text{ and }y(x)=\frac{1}{8}\sqrt{W_1}\sqrt{\frac{1}{b_2^2{W_1}^{3/2}}}\sqrt{W_2}-\frac{a_2x+c_2}{b_2},$$

where
$W_1=16\gamma_2+16\alpha_2x-3$, and
$W_2=64\,(a_2x+c_2)^{\,2}\sqrt{W_1}-2\sqrt{W_1}\left(32\,(a_2x+c_2)^{\,2}-24\gamma_2-24\alpha_2x+9\right)+\frac{2W_1\left(32(b_2y_0+c_2)^2-24\gamma_2+9\right)}{\sqrt{16\gamma_2-3}}$.

Replacing by $x=0$ in the value of $y(x)$ we get

$$y_1\text{ (or }y_2)=-\frac{\frac{b_2\sqrt{\frac{1}{b_2^2(16\gamma_2-3)^{3/2}}}\left(\sqrt{16\gamma_2-3}(b_2y_0+c_2)^2\right)^{3/2}}{(b_2y_0+c_2)^2}+c_2}{b_2},$$

and

$$y_2\text{ (or }y_1)=\frac{\frac{b_2\sqrt{\frac{1}{b_2^2(16\gamma_2-3)^{3/2}}}\left(\sqrt{16\gamma_2-3}(b_2y_0+c_2)^2\right)^{3/2}}{(b_2y_0+c_2)^2}-c_2}{b_2}.$$

Since the midpoint of this last point y_1 and y_2 is

$$(y_1+y_2)/2=-c_2/b_2, \tag{31}$$ {mp1}

and because the midpoints of the segments $[y_{1_2},y_{2_2}]$ and $[y_{1_1},y_{2_1}]$ are given, respectively, by

$$(y_{1_2}+y_{2_2})/2=\frac{\sqrt{3}b_2\sqrt{\frac{4\gamma_2\left(\frac{3\gamma_2}{(3c_1+4c_2)^2}+2\right)-3}{b_2^2}}+\frac{6\gamma_2}{3c_1+4c_2}-8c_2}{8b_2}, \tag{32}$$ {mp2}

and

$$(y_{1_1}+y_{2_1})/2=-\frac{\sqrt{3}b_2\sqrt{\frac{4\gamma_2\left(\frac{3\gamma_2}{(3c_1+4c_2)^2}+2\right)-3}{b_2^2}}-\frac{6\gamma_2}{3c_1+4c_2}+8c_2}{8b_2}. \tag{33}$$ {mp3}

Equation the right-hand sides of (31) and (32) (and (33)) we get $\gamma_2=3/8$. This value of γ_2 gives complex values for y_{1_2}, y_{2_2}, y_{1_1} and y_{2_1}. Finally, the continuous piecewise differential systems formed by (8) and (10) have no limit cycles.

Subcase 2.1.2: We consider v_8 of (29). We have now a piecewise continuous differential system formed by systems (8) and (9), we solve the algebraic system (12) we get two different pairs (y_1,y_2). The first pair is

$$y_1=\frac{-\sqrt{4b_2^2-9\beta_1^2}+b_2\left(\beta_1\sqrt{\mathcal{T}_1}-2\right)+6\beta_1c_1}{8b_2\beta_1},$$

and

$$y_2=-\frac{\sqrt{4b_2^2-9\beta_1^2}+b_2\left(\beta_1\sqrt{\mathcal{T}_1}+2\right)-6\beta_1c_1}{8b_2\beta_1}.$$

Where

$$\mathcal{T}_1=\frac{-\frac{4b_2\left(\sqrt{4b_2^2-9\beta_1^2}+2b_2\right)}{\beta_1^2}-9}{b_2^2}.$$

The second pair is

$$y_1=\frac{\sqrt{4b_2^2-9\beta_1^2}+b_2\left(\beta_1\sqrt{\mathcal{T}_2}-2\right)+6\beta_1c_1}{8b_2\beta_1},$$

and

$$y_2=\frac{\sqrt{4b_2^2-9\beta_1^2}-b_2\left(\beta_1\sqrt{\mathcal{T}_2}+2\right)+6\beta_1c_1}{8b_2\beta_1}.$$

Where

$$\mathcal{T}_2 = \frac{\frac{4b_2\left(\sqrt{4b_2^2-9\beta_1^2}-2b_2\right)}{\beta_1^2} - 9}{b_2^2}.$$

Since we have

$$\mathcal{T}_1 < 0 \text{ and } \mathcal{T}_2 < 0.$$

Then we have no limit cycles formed by systems (8) and (9).

Subcase 2.2: We consider z_6 of (28). We solve now the fourth equation of (25) to obtain one of the following sets of real solutions

$$\begin{array}{lll} u_1 & = & \{b_2 = 0\}, \\ u_2 & = & \{\alpha_1 = 0\}, \\ u_3 & = & \{\alpha_2 = \frac{\alpha_1(9-48\gamma_2)}{4(4\gamma_2-3)}\} \quad \text{and} \\ u_4 & = & \{\alpha_1 = 0,\ \gamma_2 = \frac{3}{4}\}. \end{array}$$

The allowed solution which does not contradict (11) is v_3. Then we have

$$\alpha_2 = \frac{\alpha_1\,(9-48\gamma_2)}{4\,(4\gamma_2-3)}.$$

We have now a piecewise continuous differential systems formed by systems (8) and (9), the algebraic system (12) is equivalent to

$$\frac{b_2\,(y_1-y_2)\,(-2b_2y_1-2b_2y_2+3c_1)}{(4\gamma_2-3)^{\,2}} = 0,$$

and

$$\frac{b_2\,(y_1-y_2)\,(-2b_2y_1-2b_2y_2+3c_1)\left(24b_2c_1y_1+24b_2c_1y_2-16b_2^2y_1^2-16b_2^2y_2^2-18c_1^2+24\gamma_2-9\right)}{16\gamma_2-3} = 0.$$

These two last equations have the solution $y_1 = y_2$ or $y_1 = -y_2 + 3c_1/(2b_2)$. Then we have a continuum of periodic orbits and then no limit cycles formed by systems (8) and (9).

Subcase 2.3: We consider z_8 of (28). We solve now the fourth equation of (25) to obtain one of the following sets solutions

$$\begin{array}{lll} u_1 & = & \{b_2 = 0\}, \\ u_2 & = & \{\alpha_1 = 0\} \quad \text{and} \\ u_3 & = & \{\alpha_2 = 0\}. \end{array}$$

All allowed solution contradict (11). Then we have we have no piecewise continuous differential systems formed by systems (8) and (9) in this case. ■

Proof of statement (g) of Theorem 1. In order that the piecewise differential system formed by two systems (8) and (10) be continuous they must coincide on $x = 0$; i.e. the coefficients of the system must satisfy the following algebraic system

$$
\begin{aligned}
&-3a_2b_1\beta_2\gamma_1c_1+8a_1b_2\beta_1c_2\gamma_2-3a_2b_1\beta_2c_1+3a_1b_2\beta_1c_2+4a_1\beta_1\beta_2\gamma_2^2-3a_2\beta_1\beta_2\gamma_1\\
&+3a_1\beta_1\beta_2\gamma_2+3a_2\beta_1\beta_2c_1^2-16a_1\beta_1\beta_2c_2^2-4\alpha_1b_1\beta_2\gamma_2^2+3\alpha_2b_2\beta_1\gamma_1-3\alpha_1b_1\beta_2\gamma_2\\
&-3\alpha_2b_2\beta_1c_1^2+16\alpha_1b_1\beta_2c_2^2+3\alpha_2b_1b_2\gamma_1c_1-8\alpha_1b_1b_2c_2\gamma_2+3\alpha_2b_1b_2c_1-3\alpha_1b_1b_2c_2=0,\\[4pt]
&-3a_2\beta_2b_1^2\gamma_1+8a_1b_2^2\beta_1\gamma_2-3a_2\beta_2b_1^2+3a_1b_2^2\beta_1+3a_2\beta_1\beta_2b_1c_1-24a_1b_2\beta_1\beta_2c_2+8a_1\beta_1\beta_2^2\gamma_2\\
&+3a_1\beta_1\beta_2^2-3a_2\beta_1^2\beta_2-8\alpha_1\beta_2^2b_1\gamma_2-3\alpha_1\beta_2^2b_1+3\alpha_2b_2\beta_1^2+3\alpha_2b_2b_1^2\gamma_1-8\alpha_1b_2^2b_1\gamma_2\\
&+3\alpha_2b_2b_1^2-3\alpha_1b_2^2b_1-3\alpha_2b_2\beta_1b_1c_1+24\alpha_1b_2\beta_2b_1c_2=0,\\[4pt]
&4\beta_2(\beta_2^2-2b_2^2)=0,\\[4pt]
&-4a_1\alpha_2\beta_1\gamma_2^2+3a_2\alpha_1\beta_2\gamma_1-3a_1\alpha_2\beta_1\gamma_2-3a_1\alpha_2b_2\gamma_1c_1+8a_2\alpha_1b_1c_2\gamma_2-3a_1\alpha_2b_2c_1\\
&+3a_2\alpha_1b_1c_2-3a_2\alpha_1\beta_2c_1^2+16a_1\alpha_2\beta_1c_2^2+3a_1a_2\beta_2\gamma_1c_1-8a_1a_2\beta_1c_2\gamma_2+3a_1a_2\beta_2c_1\\
&-3a_1a_2\beta_1c_2+4\alpha_1\alpha_2b_1\gamma_2^2+2a_1\alpha_2b_2\beta_1c_2+a_1a_2\beta_1\beta_2c_1-a_1a_2\beta_1\beta_2c_2-\alpha_2\alpha_1b_2\beta_1\\
&+\alpha_2\alpha_1b_1\beta_2+2\alpha_2\alpha_1b_1b_2c_1-3\alpha_1\alpha_2b_2\gamma_1+3\alpha_1\alpha_2b_1\gamma_2+3\alpha_1\alpha_2b_2c_1^2-16\alpha_1\alpha_2b_1c_2^2=0,\\[4pt]
&-8a_1\alpha_2\beta_1\beta_2\gamma_2+3a_2\alpha_1\beta_1\beta_2-3a_1\alpha_2\beta_1\beta_2+8a_2\alpha_1b_1b_2\gamma_2-3a_1\alpha_2b_1b_2\gamma_1+3a_2\alpha_1b_1b_2\\
&-3a_1\alpha_2b_1b_2+3a_1a_2b_1\beta_2\gamma_1-8a_1a_2b_2\beta_1\gamma_2-3a_1a_2b_2\beta_1+3a_1a_2b_1\beta_2-6a_2\alpha_1b_1\beta_2c_1\\
&+8a_2\alpha_1b_1\beta_2c_2-3a_1\alpha_2b_2\beta_1c_1+32a_1\alpha_2b_2\beta_1c_2+3a_1a_2\beta_1\beta_2c_1-8a_1a_2\beta_1\beta_2c_2\\
&+8\alpha_2\alpha_1b_1\beta_2\gamma_2-3\alpha_2\alpha_1b_2\beta_1+3\alpha_2\alpha_1b_1\beta_2+6\alpha_2\alpha_1b_1b_2c_1-32\alpha_2\alpha_1b_1b_2c_2=0,\\[4pt]
&8a_2\beta_2b_2-3a_2b_1\beta_2+4\alpha_2\beta_2^2-16\alpha_2b_2^2+3\alpha_2b_1b_2=0,
\end{aligned}
\tag{34}
$$

{cfpc.2.4}

together with conditions (11).

Solving the second and the third equation of (34), we get the following sets of real solutions

$$
\begin{aligned}
s_1 &= \{b_2=0,\beta_2=0\},\\
s_2 &= \{b_2=0,\beta_2=0\},\\
s_3 &= \{\beta_2=0,\gamma_1=\tfrac{-\beta_1(a_1b_2(8\gamma_2+3)+3\alpha_2\beta_1)-3\alpha_2b_1^2+b_1(\alpha_1b_2(8\gamma_2+3)+3\alpha_2\beta_1c_1)}{3\alpha_2b_1^2}\},\\
s_4 &= \{\beta_2=-\sqrt{2}b_2,\\
\gamma_1 &= -\tfrac{\beta_1(a_1b_2(8\sqrt{2}c_2+8\gamma_2+3)+\alpha_2\beta_1)+\sqrt{2}a_2(-\beta_1b_1c_1+b_1^2+\beta_1^2)+\alpha_2b_1^2-b_1(\alpha_1b_2(8\sqrt{2}c_2+8\gamma_2+3)+\alpha_2\beta_1c_1)}{b_1^2(\sqrt{2}a_2+\alpha_2)}\},\\
s_5 &= \{\beta_2=\sqrt{2}b_2,\\
\gamma_1 &= \tfrac{a_1b_2\beta_1(-8\sqrt{2}c_2+8\gamma_2+3)-\sqrt{2}a_2(-\beta_1b_1c_1+b_1^2+\beta_1^2)+\alpha_2\beta_1^2+\alpha_2b_1^2-\alpha_2\beta_1b_1c_1+\alpha_1b_2b_1(8\sqrt{2}c_2-8\gamma_2-3)}{b_1^2(\sqrt{2}a_2-\alpha_2)}\},\\
s_6 &= \{b_1=0,b_2=0,\beta_2=0\},\\
s_7 &= \{b_1=0,b_2=0,\beta_2=0\},\\
s_8 &= \{b_1=0,\alpha_2=-\tfrac{a_1b_2(8\gamma_2+3)}{3\beta_1},\beta_2=0\},\\
s_9 &= \{b_1=0,\beta_1=0,\beta_2=0\},\\
s_{10} &= \{b_1=0,c_2=\tfrac{1}{16}(-\tfrac{\beta_1(2a_2+\sqrt{2}\alpha_2)}{a_1b_2}-\sqrt{2}(8\gamma_2+3)),\beta_2=-\sqrt{2}b_2\},\\
s_{11} &= \{b_1=0,\beta_1=0,\beta_2=-\sqrt{2}b_2\},\\
s_{12} &= \{b_1=0,c_2=\tfrac{\beta_1(\alpha_2-\sqrt{2}a_2)+a_1b_2(8\gamma_2+3)}{8\sqrt{2}a_1b_2},\beta_2=\sqrt{2}b_2\},\\
s_{13} &= \{b_1=0,\beta_1=0,\beta_2=\sqrt{2}b_2\},\\
s_{14} &= \{\alpha_1=\tfrac{a_1\beta_1}{b_1},\alpha_2=0,\beta_2=0\},\\
s_{15} &= \{\alpha_2=0,\beta_2=0,\gamma_2=-\tfrac{3}{8}\},\\
s_{16} &= \{\alpha_1=\tfrac{a_1\beta_1}{b_1},\alpha_2=-\sqrt{2}a_2,\beta_2=-\sqrt{2}b_2\},\\
s_{17} &= \{c_2=-\tfrac{8\gamma_2+3}{8\sqrt{2}},\alpha_2=-\sqrt{2}a_2,\beta_2=-\sqrt{2}b_2\},\\
s_{18} &= \{\alpha_1=\tfrac{a_1\beta_1}{b_1},\alpha_2=\sqrt{2}a_2,\beta_2=\sqrt{2}b_2\},\\
s_{19} &= \{c_2=\tfrac{8\gamma_2+3}{8\sqrt{2}},\alpha_2=\sqrt{2}a_2,\beta_2=\sqrt{2}b_2\},\\
s_{20} &= \{a_1=0,b_1=0,\alpha_2=-\sqrt{2}a_2,\beta_2=-\sqrt{2}b_2\},\\
s_{21} &= \{a_1=0,b_1=0,\alpha_2=\sqrt{2}a_2,\beta_2=\sqrt{2}b_2\},\\
s_{22} &= \{a_2=0,c_2=-\tfrac{8\gamma_2+3}{8\sqrt{2}},\alpha_2=0,\beta_2=-\sqrt{2}b_2\}\quad\text{and}\\
s_{23} &= \{a_2=0,c_2=\tfrac{8\gamma_2+3}{8\sqrt{2}},\alpha_2=0,\beta_2=\sqrt{2}b_2\}.
\end{aligned}
\tag{35}
$$

{s8}

The allowed solutions which do not contradict (11) are s_3, s_4, s_5, s_8, s_{10} and s_{12} of (35). Then we have six different cases.

Case 1: We consider s_3 of (35). We solve now the first, the fifth and the sixth equation of (34) to obtain one of the following sets of real solutions

$$\begin{array}{rcl}
d_1 &=& \{b_2=0\},\\
d_2 &=& \{\alpha_1=\frac{a_1\beta_1}{b_1},\alpha_2=0\},\\
d_3 &=& \{\alpha_2=0,\gamma_2=-\frac{3}{8}\},\\
d_4 &=& \{a_1=0,b_1=0,\alpha_2=0\},\\
d_5 &=& \{b_1=0,b_2=0,\alpha_2=0\},\\
d_6 &=& \{b_1=0,b_2=0,\beta_1=0\},\\
d_7 &=& \{b_1=0,\alpha_2=0,\beta_1=0\},\\
d_8 &=& \{b_1=0,\alpha_2=0,\gamma_2=-\frac{3}{8}\},\\
d_9 &=& \{b_2=\frac{3b_1}{16},\alpha_1=\frac{a_1\beta_1}{b_1},\alpha_2=0\},\\
d_{10} &=& \{b_2=\frac{3b_1}{16},\alpha_2=\frac{b_1(8\gamma_2+3)(3b_1c_1-16b_1c_2+3\beta_1)(\alpha_1b_1-a_1\beta_1)}{48\beta_1(b_1^2+\beta_1^2)},\\
a_2 &=& \frac{b_1(b_1\beta_1(a_1(18c_1^2-192c_2c_1+512c_2^2+9)-9\alpha_1c_1+48\alpha_1c_2)+3\beta_1^2(3a_1c_1-16a_1c_2+3\alpha_1)-2\alpha_1b_1^2(3c_1-16c_2)^2)}{48\beta_1(b_1^2+\beta_1^2)}\},\\
d_{11} &=& \{b_2=\frac{3b_1}{16},\alpha_2=0,\gamma_2=-\frac{3}{8}\},\\
d_{12} &=& \{a_2=\frac{a_1b_2}{b_1},c_2=\frac{b_2(b_1c_1+\beta_1)}{b_1^2},\alpha_2=0\},\\
d_{13} &=& \{a_2=\frac{a_1b_2}{b_1},\alpha_1=\frac{a_1\beta_1}{b_1},\alpha_2=0\},\\
d_{14} &=& \{b_2=\frac{3b_1}{16},\alpha_1=0,\beta_1=0\},\\
d_{15} &=& \{a_1=0,b_1=0,b_2=0,\alpha_2=0\},\\
d_{16} &=& \{a_2=\frac{3a_1}{16},b_2=\frac{3b_1}{16},c_2=\frac{3c_1}{16},\beta_1=0\},\\
d_{17} &=& \{a_2=\frac{24a_1\gamma_2+9a_1-96\alpha_2c_1+512\alpha_2c_2}{128\gamma_2+48},b_2=\frac{3b_1}{16},\alpha_1=0,\beta_1=0\},\\
d_{18} &=& \{b_2=\frac{3b_1}{16},c_2=\frac{3c_1}{16},\beta_1=0,\gamma_2=-\frac{3}{8}\},\\
d_{19} &=& \{b_2=\frac{3b_1}{16},\alpha_2=0,\beta_1=0,\gamma_2=-\frac{3}{8}\} \quad \text{and}\\
d_{20} &=& \{a_1=0,a_2=\frac{32\alpha_2c_2-6\alpha_2c_1}{8\gamma_2+3},b_2=\frac{3b_1}{16},\alpha_1=0,\beta_1=0\}.
\end{array} \qquad (36) \quad \{\mathtt{d6}\}$$

All real solutions contradict (11) except d_{10}, d_{16} and d_{18} of (36). Then we consider these three different subcases.

Subcase 1.1: We consider d_{10} of (36). The fourth equation of (34) becomes

$$\frac{b_1\left(8\gamma_2+3\right)\left(\alpha_1b_1-a_1\beta_1\right)^2\left(b_1\left(3c_1-16c_2\right)\left(\left(3c_1-16c_2\right)^2+\left(8\gamma_2+3\right)^2\right)+48\beta_1\gamma_2\left(4\gamma_2+3\right)\right)}{768\beta_1\left(b_1^2+\beta_1^2\right)}=0,$$

which has one of the following sets of real solutions

$$\begin{array}{rcl}
z_1 &=& \{b_1=0\},\\
z_2 &=& \{\beta_1=\frac{\alpha_1b_1}{a_1}\},\\
z_3 &=& \{\beta_1=-\frac{b_1(3c_1-16c_2)(9c_1^2-96c_2c_1+256c_2^2+(8\gamma_2+3)^2)}{48\gamma_2(4\gamma_2+3)}\},\\
z_4 &=& \{\gamma_2=-\frac{3}{8}\},\\
z_5 &=& \{a_1=0,b_1=0\},\\
z_6 &=& \{a_1=0,\alpha_1=0\},\\
z_7 &=& \{b_1=0,\gamma_2=-\frac{3}{4}\},\\
z_8 &=& \{b_1=0,\gamma_2=0\},\\
z_9 &=& \{c_2=\frac{3c_1}{16},\gamma_2=-\frac{3}{4}\} \quad \text{and}\\
z_{10} &=& \{c_2=\frac{3c_1}{16},\gamma_2=0\}.
\end{array} \qquad (37) \quad \{\mathtt{z3}\}$$

All real solutions contradict (11) except z_3, z_9 and z_{10} of (37). Then we have three different subcases.

For these cases we have now a piecewise continuous differential systems formed by systems (8) and (10).

Subcase 1.1.1: We consider z_3 of (37). We solve the algebraic system (12) and we get the pair (y_1, y_2) such that

$$y_1=-\frac{16c_2}{3b_1} \quad \text{and} \quad y_2=-\frac{16c_2}{3b_1}.$$

Then we have no periodic orbits, and then no limit cycles formed by systems (8) and (10) in this case.

Subcase 1.1.2: We consider z_9 of (37). We solve the algebraic system (12) and we get the pair (y_1, y_2) such that

$$y_1 = -\frac{c_1}{b_1} \quad \text{and} \quad y_2 = -\frac{c_1}{b_1}.$$

Then we have no periodic orbits, and then no limit cycles formed by systems (8) and (10) in this case.

Subcase 1.1.3: We consider z_{10} of (37). We solve the algebraic system (12) and we get the pair (y_1, y_2) such that

$$y_1 = -\frac{c_1}{b_1} \quad \text{and} \quad y_2 = -\frac{c_1}{b_1}.$$

Then we have no periodic orbits, and then no limit cycles formed by systems (8) and (10) in this case.

Subcase 1.2: We consider d_{16} of (36). The fourth equation of (34) becomes

$$\frac{1}{256}\alpha_1 b_1 (8\gamma_2+3)(16\alpha_2(8\gamma_2+3)-9\alpha_1)=0,$$

which has one of the following sets of real solutions

$$\begin{array}{rcl} z_1 &=& \{b_1=0\}, \\ z_2 &=& \{\alpha_1=0\}, \\ z_3 &=& \{\alpha_1=\frac{16}{9}\alpha_2(8\gamma_2+3)\} \quad \text{and} \\ z_4 &=& \{\gamma_2=-\frac{3}{8}\}. \end{array} \tag{38}$$ {z4}

All real solutions contradict (11) except z_3 and z_4 of (38). Then we have three different subcases.

We have now a piecewise continuous differential systems formed by systems (8) and (10).

Subcase 1.2.1: We consider z_3 of (38). We solve the algebraic system (12) and we get

$$y_1 = -\frac{2c_1}{b_1} - y_2.$$

Then we have a continuum of periodic orbits, and then no limit cycles formed by systems (8) and (10) in this case.

Subcase 1.2.2: We consider z_4 of (38). First integrals have a singularity on the $y-$axis. Then we have no periodic orbits, and then no limit cycles formed by systems (8) and (10) in this case.

Subcase 1.3: We consider d_{18} of (36). We have immediately a piecewise continuous differential system formed by systems (8) and (10). the first integral of (10) has a singularity on the $y-$axis. Then we have no periodic orbits, and then no limit cycles formed by systems (8) and (10) in this case.

Case 2: We consider s_4 of (35). We solve now the sixth equation of (34) to get one of the following sets of real solutions

$$\begin{array}{rcl} s_1 &=& \{b_2=0\}, \\ s_2 &=& \{b_2=\frac{3b_1}{8}\} \quad \text{and} \\ s_3 &=& \{\alpha_2=-\sqrt{2}a_2\}. \end{array}$$

The only allowed real solution is s_2, we consider then $b_2 = 3\,b_1/8$. Solving the first equation of (34), this gives all the following sets of real solutions

$$\begin{array}{rcl} u_1 &=& \{\alpha_2=\frac{1}{24\beta_1(b_1^2+\beta_1^2)}[\beta_1 b_1^2(8a_1(8\gamma_2+3)c_2-9a_1c_1(8\sqrt{2}c_2+8\gamma_2+3)+128\sqrt{2}a_1c_2^2 \\ && -8\sqrt{2}a_1\gamma_2(4\gamma_2+3)-24\sqrt{2}a_2+9\alpha_1(8\sqrt{2}c_2+8\gamma_2+3))-9a_1\beta_1^2 b_1(8\sqrt{2}c_2+8\gamma_2+3) \\ && -24\sqrt{2}a_2\beta_1^3+\alpha_1 b_1^3(-8(8\gamma_2+3)c_2+9c_1(8\sqrt{2}c_2+8\gamma_2+3)-128\sqrt{2}c_2^2 \\ && +8\sqrt{2}\gamma_2(4\gamma_2+3))]\}, \\ u_2 &=& \{b_1=0,\beta_1=0\}, \\ u_3 &=& \{c_1=\frac{8(\sqrt{2}(8\gamma_2+3)c_2+32c_2^2-2\gamma_2(4\gamma_2+3))}{9(16c_2+\sqrt{2}(8\gamma_2+3))},\beta_1=0\} \quad \text{and} \\ u_4 &=& \{\alpha_1=0,\beta_1=0\}. \end{array} \tag{39}$$ {u5}

All real solutions contradict (11) except u_1 and u_3 of (39). Then we have two different subcases.

Subcase 2.1: We consider u_1 of (39). Solving the fifth equation of (34) we obtain

$$
\begin{array}{rcl}
z_1 &=& \{a_2=\frac{1}{72\beta_1(b_1^2+\beta_1^2)(16c_2+\sqrt{2}(8\gamma_2+3))}[b_1(\beta_1 b_1(a_1(512\sqrt{2}(8\gamma_2+3)c_2^2-64(16\gamma_2^2+12\gamma_2-9)c_2\\
&+& 54c_1^2(16c_2+\sqrt{2}(8\gamma_2+3))-6c_1(80\sqrt{2}(8\gamma_2+3)c_2+1024c_2^2+128\gamma_2^2+96\gamma_2+27)\\
&+& 8192c_2^3+\sqrt{2}(-512\gamma_2^3-576\gamma_2^2+72\gamma_2+81))+3\alpha_1(2(68\sqrt{2}(8\gamma_2+3)c_2+640c_2^2+224\gamma_2^2\\
&+& 168\gamma_2+27)-9c_1(16c_2+\sqrt{2}(8\gamma_2+3))))+3\beta_1^2(9a_1c_1(16c_2+\sqrt{2}(8\gamma_2+3))\\
&-& 2a_1(68\sqrt{2}(8\gamma_2+3)c_2+640c_2^2+224\gamma_2^2+168\gamma_2+27)+9\sqrt{2}\alpha_1(8\gamma_2+3)+144\alpha_1c_2)\\
&-& 2\alpha_1b_1^2(27c_1^2(16c_2+\sqrt{2}(8\gamma_2+3))-3c_1(80\sqrt{2}(8\gamma_2+3)c_2+1024c_2^2+128\gamma_2^2+96\gamma_2+27)\\
&+& 8(32\sqrt{2}(8\gamma_2+3)c_2^2+(-64\gamma_2^2-48\gamma_2+9)c_2+512c_2^3-\sqrt{2}\gamma_2(32\gamma_2^2+36\gamma_2+9))))]\},\\
z_2 &=& \{b_1=0\},\\
z_3 &=& \{\beta_1=\frac{\alpha_1 b_1}{a_1}\},\\
z_4 &=& \{a_1=0, b_1=0\},\\
z_5 &=& \{a_1=0,\alpha_1=0\},\\
z_6 &=& \{b_1=0, c_2=-\frac{8\gamma_2+3}{8\sqrt{2}}\},\\
z_7 &=& \{b_1=0,\beta_1=0\},\\
z_8 &=& \{a_1=\frac{\alpha_1 b_1}{\beta_1}, c_2=-\frac{8\gamma_2+3}{8\sqrt{2}}\},\\
z_9 &=& \{c_1=\frac{3\beta_1-\sqrt{2}b_1(8\gamma_2+3)}{6b_1}, c_2=-\frac{8\gamma_2+3}{8\sqrt{2}}\},\\
z_{10} &=& \{c_1=\frac{1}{6}\left(32c_2+\sqrt{2}\left(8\gamma_2+3\right)\right), \beta_1=0\},\\
z_{11} &=& \{c_1=\frac{8\left(\sqrt{2}(8\gamma_2+3)c_2+32c_2^2-2\gamma_2(4\gamma_2+3)\right)}{9\left(16c_2+\sqrt{2}(8\gamma_2+3)\right)}, \beta_1=0\},\\
z_{12} &=& \{\alpha_1=0,\beta_1=0\} \quad \text{and}\\
z_{13} &=& \{c_1=-\frac{8\gamma_2+3}{3\sqrt{2}}, c_2=-\frac{8\gamma_2+3}{8\sqrt{2}}, \beta_1=0\}.
\end{array}
\tag*{(40) $\texttt{\{z5\}}$}
$$

All real solutions contradict (11) except z_1 and z_9 of (40). Then we have two different subcases.

Subcase 2.1.1: We consider z_1 of (40). We solve now the fourth equation of (34) we obtain

$$
\begin{array}{rcl}
v_1 &=& \{b_1=0\},\\
v_2 &=& \{\beta_1=\frac{\alpha_1 b_1}{a_1}\},\\
v_3 &=& \{\beta_1=-[b_1(243c_1^3(16\sqrt{2}(8\gamma_2+3)c_2+128c_2^2+(8\gamma_2+3)^2)-648c_1^2(24\sqrt{2}(8\gamma_2+3)c_2^2\\
&+& 256c_2^3+9c_2-\sqrt{2}\gamma_2(32\gamma_2^2+36\gamma_2+9))+3c_1(10240\sqrt{2}(8\gamma_2+3)c_2^3+3072(4\gamma_2^2+3\gamma_2+6)c_2^2\\
&+& 8\sqrt{2}(512\gamma_2^3+576\gamma_2^2+1656\gamma_2+567)c_2+114688c_2^4+10240\gamma_2^4+15360\gamma_2^3+11808\gamma_2^2\\
&+& 4536\gamma_2+729)-8(3072\sqrt{2}(8\gamma_2+3)c_2^4+128(64\gamma_2^2+48\gamma_2+81)c_2^3+16\sqrt{2}(128\gamma_2^3+144\gamma_2^2\\
&+& 396\gamma_2+135)c_2^2-9(32\gamma_2^2+24\gamma_2-27)c_2+32768c_2^5-\sqrt{2}\gamma_2(1024\gamma_2^4+1920\gamma_2^3+2016\gamma_2^2\\
&+& 1188\gamma_2+243)))]/[24(512\sqrt{2}(8\gamma_2+3)c_2^3+24(256\gamma_2^2+192\gamma_2+33)c_2^2+\sqrt{2}(2048\gamma_2^3+2304\gamma_2^2\\
&+& 792\gamma_2+81)c_2+2048c_2^4+\gamma_2(512\gamma_2^3+768\gamma_2^2+396\gamma_2+81))]\},\\
v_4 &=& \{a_1=0, b_1=0\},\\
v_5 &=& \{a_1=0,\alpha_1=0\},\\
v_6 &=& \{b_1=0,\gamma_2=-\sqrt{2}c_2\},\\
v_7 &=& \{c_1=\frac{8c_2}{3},\gamma_2=-\sqrt{2}c_2\},\\
v_8 &=& \{b_1=0,\gamma_2=-\sqrt{2}c_2-\frac{3}{4}\} \quad \text{and}\\
v_9 &=& \{c_1=\frac{8c_2}{3},\gamma_2=-\sqrt{2}c_2-\frac{3}{4}\}.
\end{array}
\tag*{(41) $\texttt{\{v6\}}$}
$$

All real solutions contradict (11) except v_3, v_7 and v_9 of (41). Then we consider three different subcases. These cases give continuous piecewise differential systems formed by systems (8) and (10).

Subcase 2.1.1.1: We consider v_3 of (41). Now the algebraic system (12), by using only two parameters $\beta_2=-\sqrt{2}\,b_2,\ b_2=3\,b_1/8$ and the change of variables

$$y_1+y_2=t \quad \text{and} \quad y_1\,y_2=s,$$

is equivalent to the following algebraic system

$$
\begin{array}{rcl}
F_1+F_2\,s+F_3\,t &=& 0,\\
F_4\,s^2+F_5\,s+(F_6+F_7\,s)\,t^2+(F_8+F_9\,s)\,t+F_{10}\,t^3+F_{11} &=& 0.
\end{array}
\tag*{(42) $\texttt{\{ste\}}$}
$$

Where

$F_1=2(\gamma_1+1)(b_1c_1(\gamma_1+1)+\beta_1(\gamma_1-c_1^2)),$

$F_2 = 2\beta_1(-b_1\beta_1 c_1 + b_1^2(\gamma_1+1) + \beta_1^2)$,
$F_3 = b_1^2(\gamma_1+1)^2 + \beta_1^2(-c_1^2 + 2\gamma_1 + 1)$,
$F_4 = 324b_1^4(16c_2 + \sqrt{2}(8\gamma_2+3))$,
$F_5 = -18b_1^2(8\gamma_2+3)(96(8\gamma_2+3)c_2 + 256\sqrt{2}c_2^2 + \sqrt{2}(64\gamma_2(4\gamma_2+3)+45))$,
$F_6 = -18\sqrt{2}b_1^2(8\gamma_2+3)(-256c_2^2 + 32\gamma_2(4\gamma_2+3)+9)$,
$F_7 = -324b_1^4(16c_2 + \sqrt{2}(8\gamma_2+3))$,
$F_8 = 9b_1(8\gamma_2+3)^2(-256c_2^2 + 32\gamma_2(4\gamma_2+3)+9)$,
$F_9 = 54b_1^3(64\sqrt{2}(8\gamma_2+3)c_2 + 256c_2^2 + 96\gamma_2(4\gamma_2+3)+63)$,
$F_{10} = 27b_1^3(-256c_2^2 + 32\gamma_2(4\gamma_2+3)+9)$, and
$F_{11} = 16(8\gamma_2+3)^3((8\gamma_2+3)c_2 + 16\sqrt{2}c_2^2 - \sqrt{2}\gamma_2(4\gamma_2+3))$.

Solving the first equation of (42) with respect to s we get

$$s = \frac{-2b_1c_1(\gamma_1+1)^2 + b_1^2(\gamma_1+1)^2(-t) + \beta_1\left(c_1^2(2\gamma_1+\beta_1 t+2) - 2\gamma_1(\gamma_1+1) - t(2\beta_1\gamma_1+\beta_1)\right)}{2\beta_1\left(-b_1\beta_1c_1 + b_1^2(\gamma_1+1) + \beta_1^2\right)}. \quad (43)$$ {se}

We invoke parameters γ_1 from s_4 of (35), and α_2 from u_1 of (39) replacing them together with this last value of s into the second equation of (42), we get a cubic polynomial in t which has the following form

$$\left(\frac{16\left(\sqrt{2}(8\gamma_2+3)c_2 + 32c_2^2 - 2\gamma_2(4\gamma_2+3)\right)}{9b_1\left(16c_2+\sqrt{2}(8\gamma_2+3)\right)} + t\right)(G_3t^2 + G_2t + G_1) = 0. \quad (44)$$ {te}

After replacing remaining parameters, i.e.; a_2 from (40) and β_1 from (41), coefficients G_1, G_2 and G_3 are given in Appendix 4.

Solving (44) with respect to t we get

$$t = -\frac{16\left(8\sqrt{2}\gamma_2c_2 + 32c_2^2 + 3\sqrt{2}c_2 - 8\gamma_2^2 - 6\gamma_2\right)}{9b_1\left(8\sqrt{2}\gamma_2 + 16c_2 + 3\sqrt{2}\right)}, \quad (45)$$ {te1}

or

$$G_3t^2 + G_2t + G_1 = 0. \quad (46)$$ {te2}

From (45), replacing t into the first equation of (42), we obtain

$$s = \frac{64\left(8\sqrt{2}\gamma_2c_2 + 32c_2^2 + 3\sqrt{2}c_2 - 8\gamma_2^2 - 6\gamma_2\right)^2}{81b_1^2\left(8\sqrt{2}\gamma_2 + 16c_2 + 3\sqrt{2}\right)^2}.$$

and since the algebraic system $y_1 + y_2 = t$, $y_1 y_2 = s$ has a unique solution with respect to y_1 and y_2 except a permutation between y_1, y_2 which is given by

$$y_1 = \frac{1}{2}\left(t - \sqrt{t^2-4s}\right) \quad \text{and} \quad y_2 = \frac{1}{2}\left(\sqrt{t^2-4s} + t\right),$$

Then, after replacing by these last values of t and s, we get

$$y_1 = y_2 = -\frac{8\left(8\sqrt{2}\gamma_2c_2 + 32c_2^2 + 3\sqrt{2}c_2 - 8\gamma_2^2 - 6\gamma_2\right)}{9b_1\left(8\sqrt{2}\gamma_2 + 16c_2 + 3\sqrt{2}\right)}.$$

The remaining equation (46) gives together with the first equation of (42) two solutions with respect to t and s, the first is

$$t = \frac{-\sqrt{G_2^2 - 4G_1G_3} - G_2}{2G_3} \text{ and } s = \frac{-\frac{F_3\left(-\sqrt{G_2^2-4G_1G_3}-G_2\right)}{2G_3} - F_1}{F_2},$$

and the second

$$t=\frac{\sqrt{G_2^2-4G_1G_3}-G_2}{2G_3} \text{ and } s=\frac{-\frac{F_3\left(\sqrt{G_2^2-4G_1G_3}-G_2\right)}{2G_3}-F_1}{F_2}.$$

Therefore we have two pairs of (y_1,y_2), the first pair

$$y_1=-\frac{\sqrt{2}G_3\sqrt{\frac{8F_1G_3^2-4F_3\left(\sqrt{G_2^2-4G_1G_3}+G_2\right)G_3-2F_2G_1G_3+F_2\left(\sqrt{G_2^2-4G_1G_3}+G_2\right)G_2}{F_2G_3^2}}+\sqrt{G_2^2-4G_1G_3}+G_2}{4G_3},$$

and

$$y_2=-\frac{-\sqrt{2}G_3\sqrt{\frac{8F_1G_3^2-4F_3\left(\sqrt{G_2^2-4G_1G_3}+G_2\right)G_3-2F_2G_1G_3+F_2\left(\sqrt{G_2^2-4G_1G_3}+G_2\right)G_2}{F_2G_3^2}}+\sqrt{G_2^2-4G_1G_3}+G_2}{4G_3}.$$

The second pair is given as follows

$$y_1=-\frac{G_3\sqrt{\frac{2F_2\left(G_2^2-\sqrt{G_2^2-4G_1G_3}G_2-2G_1G_3\right)+8G_3\left(F_3\sqrt{G_2^2-4G_1G_3}-F_3G_2+2F_1G_3\right)}{F_2G_3^2}}-\sqrt{G_2^2-4G_1G_3}+G_2}{4G_3},$$

and

$$y_2=\frac{G_3\sqrt{\frac{2F_2\left(G_2^2-\sqrt{G_2^2-4G_1G_3}G_2-2G_1G_3\right)+8G_3\left(F_3\sqrt{G_2^2-4G_1G_3}-F_3G_2+2F_1G_3\right)}{F_2G_3^2}}+\sqrt{G_2^2-4G_1G_3}-G_2}{4G_3}.$$

Then the continuous piecewise differential systems formed by (8) and (10) can have at most two limit cycles in this case.

Numerical example with one limit cycle.
$a_1=5$, $a_2=\frac{421680\left(3688379431244448903 6\sqrt{2}-36721536410702014105\right)}{480102229040355686261577 7}$, $b_1=-1$, $b_2=-\frac{3}{8}$, $c_1=\frac{3}{4}$, $c_2=0$, $\alpha_1=10$, $\alpha_2=-\frac{1265040\left(4441267076299462989\sqrt{2}-13053293229304909636\right)}{4801022290403556862615777}$, $\beta_1=\frac{5068448\sqrt{2}+6938793}{2698752}$, $\beta_2=\frac{3}{4\sqrt{2}}$, $\gamma_1=\frac{332937\sqrt{2}+688364}{144576}$ and $\gamma_2=1$.
Figure 1 shows the crossing limit cycle in this case. Here the points of intersection with the $y-$axis are $y_1=12.7503..$ and $y_2=-1.24732...$

Subcase 2.1.1.2: We consider v_7 of (41). As in **subcase 2.1.1.1.** the continuous piecewise differential systems formed by (8) and (10) can have at most two limit cycle in this case. Indeed solving the algebraic system (12), we obtain three pairs (y_1,y_2). The first pair is

$$y_1=y_2=-\frac{8c_2}{3b_1}.$$

The second pair is

$$y_1=-\frac{3\sqrt{2}b_1^3+64\sqrt{2}c_2\beta_1b_1^2-21\beta_1b_1^2+3\sqrt{2}\sqrt{\mathcal{S}}\beta_1(2\sqrt{2}b_1+\beta_1)b_1^2-6\sqrt{2}\beta_1^2b_1+32c_2\beta_1^2b_1+3\sqrt{\mathcal{S}_1}}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)},$$

$$y_2=\frac{-3\sqrt{2}b_1^3-64\sqrt{2}c_2\beta_1b_1^2+21\beta_1b_1^2+3\sqrt{2}\sqrt{\mathcal{S}}\beta_1(2\sqrt{2}b_1+\beta_1)b_1^2+6\sqrt{2}\beta_1^2b_1-32c_2\beta_1^2b_1-3\sqrt{\mathcal{S}_1}}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)}.$$

And the third pair is

$$y_1=\frac{-3\sqrt{2}b_1^3-3\sqrt{2}\beta_1(2\sqrt{2}b_1+\beta_1)\sqrt{\mathcal{R}}b_1^2-64\sqrt{2}c_2\beta_1b_1^2+21\beta_1b_1^2+6\sqrt{2}\beta_1^2b_1-32c_2\beta_1^2b_1+3\sqrt{\mathcal{S}_1}}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)},$$

$$y_2=\frac{-3\sqrt{2}b_1^3-64\sqrt{2}c_2\beta_1b_1^2+21\beta_1b_1^2+3\sqrt{2}\sqrt{\mathcal{R}}\beta_1(2\sqrt{2}b_1+\beta_1)b_1^2+6\sqrt{2}\beta_1^2b_1-32c_2\beta_1^2b_1+3\sqrt{\mathcal{S}_1}}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)},$$

with

$$\mathcal{R}_1=-b_1^2(-2b_1^4+14\sqrt{2}\beta_1b_1^3+39\beta_1^2b_1^2+16\sqrt{2}\beta_1^3b_1+4\beta_1^4),$$

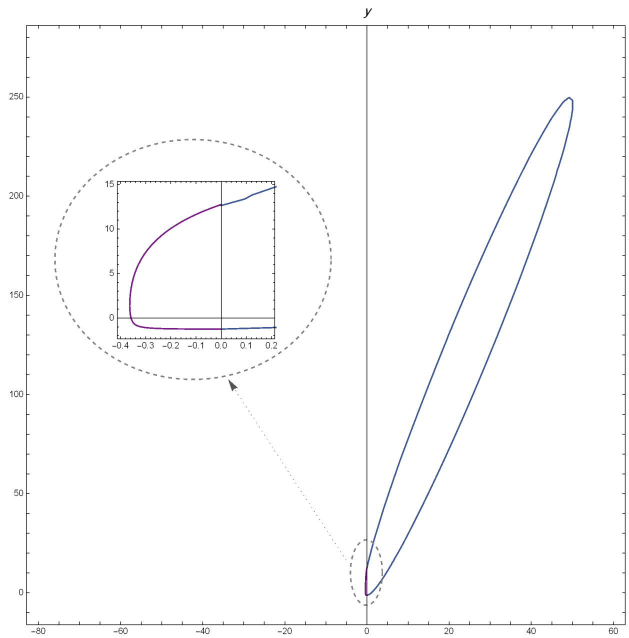

FIGURE 1. The crossing limit cycle of systems (8) and (10). The picture on the left is a zoom of the limit cycle in a neighborhood of the y-axis. {fig1}

$$\mathcal{S}_1 = b_1^2(2b_1^4 - 14\sqrt{2}\beta_1 b_1^3 - 39\beta_1^2 b_1^2 - 16\sqrt{2}\beta_1^3 b_1 - 4\beta_1^4),$$

$$\mathcal{S} = \frac{-6b_1^5 + 12\sqrt{2}\beta_1 b_1^4 + 31\beta_1^2 b_1^3 + \sqrt{2}(10\beta_1^3 - 3\sqrt{\mathcal{R}_1})b_1^2 + (2\beta_1^4 - 9\beta_1\sqrt{\mathcal{R}_1})b_1 - 2\sqrt{2}\beta_1^2\sqrt{\mathcal{R}_1}}{b_1^3\beta_1^2(2\sqrt{2}b_1 + \beta_1)^2}$$

and

$$\mathcal{R} = \frac{-6b_1^5 + 12\sqrt{2}\beta_1 b_1^4 + 31\beta_1^2 b_1^3 + \sqrt{2}(10\beta_1^3 + 3\sqrt{\mathcal{R}_1})b_1^2 + (2\beta_1^4 + 9\sqrt{\mathcal{R}_1}\beta_1)b_1 + 2\sqrt{2}\sqrt{\mathcal{R}_1}\beta_1^2}{b_1^3\beta_1^2(2\sqrt{2}b_1 + \beta_1)^2}.$$

Since the invariant straight lines for the generalized systems (8) and (10) are given respectively as follows

$$y = -\frac{8c_2}{3b_1} - \frac{\alpha_1 x + 1}{\beta_1},$$

which intersects the $y-$axis at the point $y_{01} = -\frac{8c_2}{3b_1} - \frac{1}{\beta_1}$, and

$$y = \frac{1}{6}\left(\frac{3\sqrt{2} - 16c_2}{b_1} - \frac{3x\left(2a_1b_1 + \sqrt{2}a_1\beta_1 + 2\alpha_1\beta_1 - \sqrt{2}\alpha_1 b_1\right)}{b_1^2 + \beta_1^2}\right),$$

which intersects the $y-$axis at the point $y_{02} = \frac{3\sqrt{2}-16c_2}{6b_1}$. In both pairs of $y_1,\ y_2$ give $y_1 < y_2$, and in addition we have

- If $y_{01} < y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the second pair (y_1, y_2);
- If $y_{01} > y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the third pair (y_1, y_2);
- If $y_{01} = y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the second and the third pair of $y_1,\ y_2$.

Then the continuous piecewise differential systems formed by (8) and (10) can have at most one limit cycle.

Subcase 2.1.1.3: We consider v_9 of (41). As in **subcase 2.1.1.1.** the continuous piecewise differential systems formed by (8) and (10) can have at most two limit cycle in this case. Solving the algebraic system

(12), we obtain three pairs (y_1, y_2). The first pair is

$$y_1 = y_2 = -\frac{8c_2}{3b_1}.$$

The second pair is

$$y_1 = -\frac{\sqrt{2}\sqrt{\mathcal{S}_1}}{4} - \frac{3\sqrt{\mathcal{R}_1} + \beta_1 b_1^2(64\sqrt{2}c_2 + 21) - 2\beta_1^2 b_1(16c_2 + 3\sqrt{2}) + 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1 - \beta_1)},$$

$$y_2 = \frac{\sqrt{\mathcal{S}_1}}{2\sqrt{2}} - \frac{3\sqrt{\mathcal{R}_1} + \beta_1 b_1^2(64\sqrt{2}c_2 + 21) - 2\beta_1^2 b_1(16c_2 + 3\sqrt{2}) + 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1 - \beta_1)}.$$

And the third pair is

$$y_1 = \frac{-3\sqrt{2}\beta_1 b_1^2(2\sqrt{2}b_1 - \beta_1)\sqrt{\mathcal{S}_2} - 21\beta_1 b_1^2 + 6\sqrt{2}\beta_1^2 b_1 + 3\sqrt{\mathcal{R}_1} - 64\sqrt{2}\beta_1 b_1^2 c_2 + 32\beta_1^2 b_1 c_2 - 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1 - \beta_1)},$$

$$y_2 = \frac{3\sqrt{2}\beta_1 b_1^2\sqrt{\mathcal{S}_2}(2\sqrt{2}b_1 - \beta_1) - 21\beta_1 b_1^2 + 6\sqrt{2}\beta_1^2 b_1 + 3\sqrt{\mathcal{R}_1} - 64\sqrt{2}\beta_1 b_1^2 c_2 + 32\beta_1^2 b_1 c_2 - 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1 - \beta_1)},$$

with

$$\mathcal{R}_1 = b_1^2(14\sqrt{2}\beta_1 b_1^3 - 39\beta_1^2 b_1^2 + 16\sqrt{2}\beta_1^3 b_1 + 2b_1^4 - 4\beta_1^4),$$

$$\mathcal{S}_1 = -\frac{12\sqrt{2}\beta_1 b_1^4 - 31\beta_1^2 b_1^3 + \sqrt{2}b_1^2(3\sqrt{\mathcal{R}_1} + 10\beta_1^3) - 2\beta_1^4 b_1 - 9\beta_1 b_1\sqrt{\mathcal{R}_1} + 2\sqrt{2}\beta_1^2\sqrt{\mathcal{R}_1} + 6b_1^5}{b_1^3\beta_1^2(\beta_1 - 2\sqrt{2}b_1)^2},$$

and

$$\mathcal{S}_2 = \frac{-12\sqrt{2}\beta_1 b_1^4 + 31\beta_1^2 b_1^3 + \sqrt{2}b_1^2(3\sqrt{\mathcal{R}_1} - 10\beta_1^3) + 2\beta_1^4 b_1 - 9\beta_1 b_1\sqrt{\mathcal{R}_1} + 2\sqrt{2}\beta_1^2\sqrt{\mathcal{R}_1} - 6b_1^5}{b_1^3\beta_1^2(\beta_1 - 2\sqrt{2}b_1)^2}.$$

The invariant straight lines for the generalized systems (8) and (10) are given respectively by

$$y = -\frac{8c_2}{3b_1} - \frac{\alpha_1 x + 1}{\beta_1},$$

which intersects the $y-$axis at the point $y_{01} = -\frac{8c_2}{3b_1} - \frac{1}{\beta_1}$, and

$$y = -\frac{x\left(2a_1b_1 - \sqrt{2}a_1\beta_1 + 2\alpha_1\beta_1 + \sqrt{2}\alpha_1 b_1\right)}{2\left(b_1^2 + \beta_1^2\right)} - \frac{16c_2 + 3\sqrt{2}}{6b_1},$$

which intersects the $y-$axis at the point $y_{02} = -\frac{16c_2+3\sqrt{2}}{6b_1}$. The third pair (y_1, y_2) give $y_1 < y_2$, and in addition we have

- If $y_{01} < y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the third pair (y_1, y_2);
- If $y_{01} > y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the secondd pair (y_1, y_2);
- If $y_{01} = y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the second and the third pair of $y_1,\ y_2$.

Then the continuous piecewise differential systems formed by (8) and (10) can have at most one limit cycle.

Subcase 2.1.2: We consider z_9 of (40). We solve now the fourth equation of (34) becomes

$$-9\sqrt{2}\left(\alpha_1 b_1 - a_1\beta_1\right)\left(b_1^2\left(6a_1\beta_1 - 32a_2\beta_1\right) + \left(3a_1 - 32a_2\right)\beta_1^3 + 9\alpha_1\beta_1^2 b_1 + 6\alpha_1 b_1^3\right) = 0.$$

All sets of real solution for this last equation is given as follows

$$\begin{array}{rcl} e_1 & = & \{\alpha_1 = \frac{32a_2\beta_1\left(b_1^2+\beta_1^2\right) - 3a_1\left(2b_1^2\beta_1+\beta_1^3\right)}{9\beta_1^2 b_1 + 6b_1^3}\}, \\ e_2 & = & \{\beta_1 = \frac{\alpha_1 b_1}{a_1}\}, \\ e_3 & = & \{a_1 = 0, b_1 = 0\}, \\ e_4 & = & \{a_1 = 0, \alpha_1 = 0\}, \\ e_5 & = & \{a_2 = \frac{3a_1}{32}, b_1 = 0\} \quad \text{and} \\ e_6 & = & \{b_1 = 0, \beta_1 = 0\}. \end{array} \qquad (47) \quad \{e8\}$$

The only allowed solution is e_1. Then we have

$$\alpha_1 = \frac{32\, a_2 \beta_1 \left(b_1^2 + \beta_1^2\right) - 3\, a_1 \left(2\, b_1^2 \beta_1 + \beta_1^3\right)}{9\, \beta_1^2 b_1 + 6\, b_1^3}.$$

In this case the algebraic system (12) has no real solutions

$$y_1 = \frac{\sqrt{2}\, b_1 \left(8\gamma_2 + 3\right) + (3 - 3i)\beta_1}{6 b_1^2} \quad \text{and} \quad y_2 = \frac{\sqrt{2}\, b_1 \left(8\gamma_2 + 3\right) + (3 + 3i)\beta_1}{6 b_1^2}$$

and then the continuous piecewise differential systems formed by (8) and (10) has no limit cycle in this case.

Subcase 2.2: We consider u_3 of (39). Solving the fifth equation of (34) we obtain all sets of real solutions as follows

$$\begin{aligned}
w_1 &= \{b_1 = 0\},\\
w_2 &= \{\alpha_1 = 0\},\\
w_3 &= \{\alpha_2 = \tfrac{8a_2\left(32(8\gamma_2+3)c_2+128\sqrt{2}c_2^2+\sqrt{2}\left(64\gamma_2^2+48\gamma_2+27\right)\right)-27a_1\left(32(8\gamma_2+3)c_2+128\sqrt{2}c_2^2+\sqrt{2}(8\gamma_2+3)^2\right)}{16\left(64\sqrt{2}(8\gamma_2+3)c_2+512c_2^2+256\gamma_2^2+192\gamma_2+27\right)}\}, \qquad (48) \quad \{\mathtt{w8}\}\\
w_4 &= \{a_2 = \tfrac{3a_1}{8}, c_2 = -\tfrac{16\gamma_2+9}{16\sqrt{2}}\} \quad \text{and}\\
w_5 &= \{a_2 = \tfrac{3a_1}{8}, c_2 = -\tfrac{16\gamma_2+3}{16\sqrt{2}}\}.
\end{aligned}$$

The allowed solutions are w_3, w_4 and w_5. Then we have three different subcases

Subcase 2.2.1: We consider w_3 of (48). We solve now the fourth equation of (34) and we obtain

$$\begin{aligned}
e_1 &= \{b_1 = 0\},\\
e_2 &= \{\alpha_1 = 0\},\\
e_3 &= \{\alpha_1 = -[(3a_1 - 8a_2)(8192(8\gamma_2 + 3)c_2^3 + 192\sqrt{2}(256\gamma_2^2 + 192\gamma_2 + 87)c_2^2 + 16(2048\gamma_2^3 + 2304\gamma_2^2\\
&+ 2088\gamma_2 + 567)c_2 + 16384\sqrt{2}c_2^4 + \sqrt{2}(4096\gamma_2^4 + 6144\gamma_2^3 + 8352\gamma_2^2 + 4536\gamma_2 + 729))]\\
&/ [54(1536(8\gamma_2 + 3)c_2^2 + 24\sqrt{2}(256\gamma_2^2 + 192\gamma_2 + 33)c_2 + 4096\sqrt{2}c_2^3 + 2048\gamma_2^3 + 2304\gamma_2^2 + 792\gamma_2 + 81)]\},\\
e_4 &= \{\gamma_2 = -\sqrt{2}c_2 - \tfrac{3}{8}\},\\
e_5 &= \{a_2 = \tfrac{3a_1}{8}, \gamma_2 = -\sqrt{2}c_2 - \tfrac{9}{16}\} \quad \text{and}\\
e_6 &= \{a_2 = \tfrac{3a_1}{8}, \gamma_2 = -\sqrt{2}c_2 - \tfrac{3}{16}\}.
\end{aligned}$$

The allowed solution is e_3. The algebraic system (12) in this case has complex solutions and then the continuous piecewise differential systems formed by (8) and (10) cannot have limit cycles.

Subcase 2.2.2: We consider w_4 of (48). We solve now the fourth equation of (34) which becomes

$$27\alpha_1 b_1 \left(3\sqrt{2}a_1 + 6\alpha_1 + 8\alpha_2\right) = 0,$$

we obtain

$$\begin{aligned}
e_1 &= \{b_1 = 0\},\\
e_2 &= \{\alpha_1 = 0\} \quad \text{and}\\
e_3 &= \{\alpha_2 = \tfrac{-3}{8}\left(\sqrt{2}a_1 + 2\alpha_1\right)\}.
\end{aligned}$$

The allowed solution is e_3. The algebraic system (12) in this case gives $y_1 = y_2 = \frac{4\sqrt{2}\gamma_2+3\sqrt{2}}{3b_1}$ or has complex solutions

$$y_1 = \frac{8\sqrt{2}\gamma_2 + 6\sqrt{2} - 3i\sqrt{10}}{6b_1} \quad \text{and} \quad y_2 = \frac{8\sqrt{2}\gamma_2 + 6\sqrt{2} + 3i\sqrt{10}}{6b_1},$$

and then the continuous piecewise differential systems formed by (8) and (10) cannot have limit cycles.

Subcase 2.2.3: We consider w_5 of (48). We solve now the fourth equation of (34) which becomes

$$27\alpha_1 b_1 \left(3\sqrt{2}a_1 - 6\alpha_1 + 8\alpha_2\right) = 0,$$

we obtain

$$\begin{aligned}
e_1 &= \{b_1 = 0\},\\
e_2 &= \{\alpha_1 = 0\} \quad \text{and}\\
e_3 &= \{\alpha_2 = \tfrac{-3}{8}\left(\sqrt{2}a_1 - 2\alpha_1\right)\}.
\end{aligned}$$

The allowed solution is e_3. The algebraic system (12) in this case gives $y_1 = y_2 = \frac{4\sqrt{2}\gamma_2}{3b_1}$, or has complex solutions

$$y_1 = \frac{8\sqrt{2}\gamma_2 - 3i\sqrt{10}}{6b_1} \quad \text{and} \quad y_2 = \frac{8\sqrt{2}\gamma_2 + 3i\sqrt{10}}{6b_1},$$

and then the continuous piecewise differential systems formed by (8) and (10) cannot have limit cycles.

Case 3: We consider s_5 of (35). This case gives the same results as in the **case 2** and all bifurcated cases given in **case 2** have the same conclusion as follows:

We solve the sixth equation of (34) to get one of the following sets of real solutions

$$\begin{aligned} s_1 &= \{b_2 = 0\}, \\ s_2 &= \{b_2 = \tfrac{3b_1}{8}\} \quad \text{and} \\ s_3 &= \{\alpha_2 = \sqrt{2}a_2\}. \end{aligned}$$

The only allowed real solution is s_2, then $b_2 = 3\,b_1/8$. Solving the first equation of (34), this gives all the following sets of real solutions

$$\begin{aligned} u_1 &= \{\alpha_2 = \tfrac{1}{24\beta_1(b_1^2+\beta_1^2)}[9a_1\beta_1^2 b_1(8\sqrt{2}c_2 - 8\gamma_2 - 3) + 24\sqrt{2}a_2\beta_1^3 - \alpha_1 b_1^3(9c_1(8\sqrt{2}c_2 - 8\gamma_2 - 3) \\ &\quad +8((8\gamma_2+3)c_2 - 16\sqrt{2}c_2^2 + \sqrt{2}\gamma_2(4\gamma_2+3))) \\ &\quad b_1^2\beta_1(9a_1c_1(8\sqrt{2}c_2 - 8\gamma_2 - 3) + 8a_1((8\gamma_2+3)c_2 - 16\sqrt{2}c_2^2 + \sqrt{2}\gamma_2(4\gamma_2+3)) \\ &\quad +24\sqrt{2}a_2 + 9\alpha_1(-8\sqrt{2}c_2 + 8\gamma_2 + 3))]\}, \\ u_2 &= \{b_1 = 0, \beta_1 = 0\}, \\ u_3 &= \{c_1 = \tfrac{8(\sqrt{2}(8\gamma_2+3)c_2 - 32c_2^2 + 2\gamma_2(4\gamma_2+3))}{9(\sqrt{2}(8\gamma_2+3) - 16c_2)}, \beta_1 = 0\} \quad \text{and} \\ u_4 &= \{\alpha_1 = 0, \beta_1 = 0\}. \end{aligned} \tag{49}$$
{u6}

All real solutions contradict (11) except u_1 and u_3 of (49). Then we have two different subcases.

Subcase 3.1: We consider u_1 of (49). Solving the fifth equation of (34) we obtain

$$\begin{aligned} z_1 &= \{a_2 = \tfrac{1}{72\beta_1(b_1^2+\beta_1^2)(\sqrt{2}(8\gamma_2+3)-16c_2)}[b_1(\beta_1 b_1(a_1(512\sqrt{2}(8\gamma_2+3)c_2^2 + 64(16\gamma_2^2 + 12\gamma_2 - 9)c_2 \\ &+ \ 54c_1^2(\sqrt{2}(8\gamma_2+3) - 16c_2) - 6c_1(80\sqrt{2}(8\gamma_2+3)c_2 - 1024c_2^2 - 128\gamma_2^2 - 96\gamma_2 - 27) \\ &- \ 8192c_2^3 + \sqrt{2}(-512\gamma_2^3 - 576\gamma_2^2 + 72\gamma_2 + 81)) - 3\alpha_1(-136\sqrt{2}(8\gamma_2+3)c_2 \\ &+ \ 9c_1(\sqrt{2}(8\gamma_2+3) - 16c_2) + 1280c_2^2 + 448\gamma_2^2 + 336\gamma_2 + 54)) + 3\beta_1^2(a_1(-136\sqrt{2}(8\gamma_2+3)c_2 \\ &+ \ 9c_1(\sqrt{2}(8\gamma_2+3) - 16c_2) + 1280c_2^2 + 448\gamma_2^2 + 336\gamma_2 + 54) + 9\alpha_1(\sqrt{2}(8\gamma_2+3) - 16c_2)) \\ &- \ 2\alpha_1 b_1^2(27c_1^2(\sqrt{2}(8\gamma_2+3) - 16c_2) + c_1(-240\sqrt{2}(8\gamma_2+3)c_2 + 3072c_2^2 + 384\gamma_2^2 + 288\gamma_2 + 81) \\ &- \ 8(-32\sqrt{2}(8\gamma_2+3)c_2^2 + (-64\gamma_2^2 - 48\gamma_2 + 9)c_2 + 512c_2^3 + \sqrt{2}\gamma_2(32\gamma_2^2 + 36\gamma_2 + 9))))]\}, \\ z_2 &= \{b_1 = 0\}, \\ z_3 &= \{\beta_1 = \tfrac{\alpha_1 b_1}{a_1}\}, \\ z_4 &= \{a_1 = 0, b_1 = 0\}, \\ z_5 &= \{a_1 = 0, \alpha_1 = 0\}, \\ z_6 &= \{b_1 = 0, c_2 = \tfrac{8\gamma_2+3}{8\sqrt{2}}\}, \\ z_7 &= \{b_1 = 0, \beta_1 = 0\}, \\ z_8 &= \{a_1 = \tfrac{\alpha_1 b_1}{\beta_1}, c_2 = \tfrac{8\gamma_2+3}{8\sqrt{2}}\}, \\ z_9 &= \{c_1 = \tfrac{3\beta_1 + \sqrt{2}b_1(8\gamma_2+3)}{6b_1}, c_2 = \tfrac{8\gamma_2+3}{8\sqrt{2}}\}, \\ z_{10} &= \{c_1 = \tfrac{1}{6}\left(32c_2 - \sqrt{2}\,(8\gamma_2+3)\right), \beta_1 = 0\}, \\ z_{11} &= \{c_1 = \tfrac{8\left(\sqrt{2}(8\gamma_2+3)c_2 - 32c_2^2 + 2\gamma_2(4\gamma_2+3)\right)}{9\left(\sqrt{2}(8\gamma_2+3) - 16c_2\right)}, \beta_1 = 0\}, \\ z_{12} &= \{\alpha_1 = 0, \beta_1 = 0\} \quad \text{and} \\ z_{13} &= \{c_1 = \tfrac{8\gamma_2+3}{3\sqrt{2}}, c_2 = \tfrac{8\gamma_2+3}{8\sqrt{2}}, \beta_1 = 0\}. \end{aligned} \tag{50}$$
{z6}

All real solutions contradict (11) except z_1 and z_9 of (50). Then we have two different subcases.

Subcase 3.1.1: We consider z_1 of (50). We solve now the fourth equation of (34) we obtain

$$\begin{array}{rcl}
v_1 &=& \{b_1=0\},\\
v_2 &=& \{\beta_1=\frac{\alpha_1 b_1}{a_1}\},\\
v_3 &=& \{\beta_1=-[b_1(-243c_1^3(16\sqrt{2}(8\gamma_2+3)c_2-128c_2^2-(8\gamma_2+3)^2)-648c_1^2(-24\sqrt{2}(8\gamma_2+3)c_2^2\\
&+& 256c_2^3+9c_2+\sqrt{2}\gamma_2(32\gamma_2^2+36\gamma_2+9))-3c_1(10240\sqrt{2}(8\gamma_2+3)c_2^3-3072(4\gamma_2^2\\
&+& 3\gamma_2+6)c_2^2+8\sqrt{2}(512\gamma_2^3+576\gamma_2^2+1656\gamma_2+567)c_2-114688c_2^4-10240\gamma_2^4-15360\gamma_2^3\\
&-& 11808\gamma_2^2-4536\gamma_2-729)-8(-3072\sqrt{2}(8\gamma_2+3)c_2^4+128(64\gamma_2^2+48\gamma_2+81)c_2^3\\
&-& 16\sqrt{2}(128\gamma_2^3+144\gamma_2^2+396\gamma_2+135)c_2^2-9(32\gamma_2^2+24\gamma_2-27)c_2+32768c_2^5+\sqrt{2}\gamma_2(1024\gamma_2^4\\
&+& 1920\gamma_2^3+2016\gamma_2^2+1188\gamma_2+243)))]/[24(512\sqrt{2}(8\gamma_2+3)c_2^3-24(256\gamma_2^2\\
&+& 192\gamma_2+33)c_2^2+\sqrt{2}(2048\gamma_2^3+2304\gamma_2^2+792\gamma_2+81)c_2-2048c_2^4-\gamma_2(512\gamma_2^3+768\gamma_2^2+396\gamma_2+81))]\},\\
v_4 &=& \{a_1=0,b_1=0\},\\
v_5 &=& \{a_1=0,\alpha_1=0\},\\
v_6 &=& \{b_1=0,\gamma_2=\sqrt{2}c_2\},\\
v_7 &=& \{c_1=\frac{8c_2}{3},\gamma_2=\sqrt{2}c_2\},\\
v_8 &=& \{b_1=0,\gamma_2=\sqrt{2}c_2-\frac{3}{4}\} \quad \text{and}\\
v_9 &=& \{c_1=\frac{8c_2}{3},\gamma_2=\sqrt{2}c_2-\frac{3}{4}\}.
\end{array} \tag{51}$$

All real solutions contradict (11) except v_3, v_7 and v_9 of (51). Then we consider three different subcases. These cases give continuous piecewise differential systems formed by systems (8) and (10).

Subcase 3.1.1.1: We consider v_3 of (51). Now the algebraic system (12), by using only two parameters $\beta_2=\sqrt{2}\,b_2,\ b_2=3\,b_1/8$ and the change of variables

$$y_1+y_2=t \quad \text{and} \quad y_1\,y_2=s,$$

is equivalent to the following algebraic system

$$\begin{array}{rcl}
E_1+E_2\,s+E_3\,t &=& 0,\\
E_4\,s^2+E_5\,s+(E_6+E_7\,s)\,t^2+(E_8+E_9\,s)\,t+E_{10}\,t^3+E_{11} &=& 0.
\end{array} \tag{52}$$

Where

$E_1=2\left(\gamma_1+1\right)\left(b_1c_1\left(\gamma_1+1\right)+\beta_1\left(\gamma_1-c_1^2\right)\right)$,

$E_2=2\beta_1\left(-b_1\beta_1c_1+b_1^2\left(\gamma_1+1\right)+\beta_1^2\right)$,

$E_3=b_1^2\left(\gamma_1+1\right)^2+\beta_1^2\left(-c_1^2+2\gamma_1+1\right)$,

$E_4=324b_1^4\left(\sqrt{2}\left(8\gamma_2+3\right)-16c_2\right)$,

$E_5=-18b_1^2\left(8\gamma_2+3\right)\left(-96\left(8\gamma_2+3\right)c_2+256\sqrt{2}c_2^2+\sqrt{2}\left(64\gamma_2\left(4\gamma_2+3\right)+45\right)\right)$,

$E_6=-18\sqrt{2}b_1^2\left(8\gamma_2+3\right)\left(-256c_2^2+32\gamma_2\left(4\gamma_2+3\right)+9\right)$,

$E_7=-324b_1^4\left(\sqrt{2}\left(8\gamma_2+3\right)-16c_2\right)$,

$E_8=-9b_1\left(8\gamma_2+3\right)^2\left(-256c_2^2+32\gamma_2\left(4\gamma_2+3\right)+9\right)$,

$E_9=54b_1^3\left(64\sqrt{2}\left(8\gamma_2+3\right)c_2-256c_2^2-96\gamma_2\left(4\gamma_2+3\right)-63\right)$,

$E_{10}=27b_1^3\left(256c_2^2-32\gamma_2\left(4\gamma_2+3\right)-9\right)$, and

$E_{11}=-16\left(8\gamma_2+3\right)^3\left(\left(8\gamma_2+3\right)c_2-16\sqrt{2}c_2^2+\sqrt{2}\gamma_2\left(4\gamma_2+3\right)\right)$.

Solving the first equation of (52) with respect to s we get

$$s=\frac{-2b_1c_1\left(\gamma_1+1\right)^2+b_1^2\left(\gamma_1+1\right)^2(-t)+\beta_1\left(c_1^2\left(2\gamma_1+\beta_1t+2\right)-2\gamma_1\left(\gamma_1+1\right)-t\left(2\beta_1\gamma_1+\beta_1\right)\right)}{2\beta_1\left(-b_1\beta_1c_1+b_1^2\left(\gamma_1+1\right)+\beta_1^2\right)}. \tag{53}$$

We invoke parameters γ_1 from s_5 of (35), and α_2 from u_1 of (49) replacing them together with this last value of s into the second equation of (52), we get a cubic polynomial in t which has the following form

$$\left(\frac{16\left(8\sqrt{2}\gamma_2 c_2-32c_2^2+3\sqrt{2}c_2+8\gamma_2^2+6\gamma_2\right)}{9b_1\left(8\sqrt{2}\gamma_2-16c_2+3\sqrt{2}\right)}+t\right)(H_3t^2+H_2t+H_1)=0. \tag{54}$$ {te0}

After replacing remaining parameters, i.e.; a_2 from (50) and β_1 from (51), coefficients H_1, H_2 and H_3 are given in Appendix 5.

Solving (54) with respect to t we get

$$t=-\frac{16\left(8\sqrt{2}\gamma_2 c_2-32c_2^2+3\sqrt{2}c_2+8\gamma_2^2+6\gamma_2\right)}{9b_1\left(8\sqrt{2}\gamma_2-16c_2+3\sqrt{2}\right)}, \tag{55}$$ {te01}

or

$$H_3t^2+H_2t+H_1=0. \tag{56}$$ {te02}

Replacing t, from (55), into the first equation of (52), we obtain

$$s=\frac{64\left(8\sqrt{2}\gamma_2 c_2-32c_2^2+3\sqrt{2}c_2+8\gamma_2^2+6\gamma_2\right)^2}{81b_1^2\left(8\sqrt{2}\gamma_2-16c_2+3\sqrt{2}\right)^2}.$$

and since the algebraic system $y_1+y_2=t$, $y_1\,y_2=s$ has a unique solution with respect to y_1 and y_2 except a permutation between $y_1,\ y_2$ which is given by

$$y_1=\frac{1}{2}\left(t-\sqrt{t^2-4s}\right)\quad \text{and} \quad y_2=\frac{1}{2}\left(\sqrt{t^2-4s}+t\right),$$

Then, after replacing by these last values of t and s, we get

$$y_1=y_2=\frac{8\left(8\sqrt{2}\gamma_2 c_2-32c_2^2+3\sqrt{2}c_2+8\gamma_2^2+6\gamma_2\right)}{9\left(16b_1c_2-8\sqrt{2}b_1\gamma_2-3\sqrt{2}b_1\right)}.$$

The remaining equation (56) gives together with the first equation of (52) two solutions with respect to t and s, the first is

$$t=\frac{-\sqrt{H_2^2-4H_1H_3}-H_2}{2H_3}\ \text{and}\ s=\frac{-\frac{E_3\left(-\sqrt{H_2^2-4H_1H_3}-H_2\right)}{2H_3}-E_1}{E_2},$$

and the second

$$t=\frac{\sqrt{H_2^2-4H_1H_3}-H_2}{2H_3}\ \text{and}\ s=\frac{-\frac{E_3\left(\sqrt{H_2^2-4H_1H_3}-H_2\right)}{2H_3}-E_1}{E_2}.$$

Therefore we have two pairs of (y_1,y_2), the first pair

$$y_1=-\frac{\sqrt{H_2^2-4H_1H_3}+H_2+\sqrt{2}\sqrt{\frac{8E_1H_3^2-4E_3\left(\sqrt{H_2^2-4H_1H_3}+H_2\right)H_3-2E_2H_1H_3+E_2\left(\sqrt{H_2^2-4H_1H_3}+H_2\right)H_2}{E_2H_3^2}}H_3}{4H_3},$$

and

$$y_2=-\frac{\sqrt{H_2^2-4H_1H_3}-\sqrt{2}H_3\sqrt{\frac{8E_1H_3^2-4E_3\left(\sqrt{H_2^2-4H_1H_3}+H_2\right)H_3-2E_2H_1H_3+E_2\left(\sqrt{H_2^2-4H_1H_3}+H_2\right)H_2}{E_2H_3^2}}+H_2}{4H_3}.$$

The second pair is given as follows

$$y_1=-\frac{-\sqrt{H_2^2-4H_1H_3}+H_2+\sqrt{\frac{2E_2\left(H_2^2-\sqrt{H_2^2-4H_1H_3}H_2-2H_1H_3\right)+8\left(E_3\sqrt{H_2^2-4H_1H_3}-E_3H_2+2E_1H_3\right)H_3}{E_2H_3^2}}H_3}{4H_3},$$

and

$$y_2 = \frac{\sqrt{H_2^2-4H_1H_3}-H_2+\sqrt{\frac{2E_2\left(H_2^2-\sqrt{H_2^2-4H_1H_3}H_2-2H_1H_3\right)+8\left(E_3\sqrt{H_2^2-4H_1H_3}-E_3H_2+2E_1H_3\right)H_3}{E_2H_3^2}}H_3}{4H_3}.$$

Then the continuous piecewise differential systems formed by (8) and (10) can have at most two limit cycles in this case.

Remark. Let $(0,y_1)$ and $(0,y_2)$ be with $y_1<y_2$ the two points where a limit cycle intersects the discontinuity line $x=0$ for a continuous piecewise diferential system (8)+(10). In the cases 2.1.1.1 and 3.1.1.1 we have found an upper bound of two limit cicles. But all the computations that we have done show that when we have two solutions $(0,y_{1_1})$ and $(0,y_{2_1})$, and $(0,y_{1_2})$ and $(0,y_{2_2})$, they satisfy $y_{1_1}<y_{2_1}<y_{1_2}<y_{2_2}$. This implies that these two possible limit cicles cannot surround the same equilibrium point of the continuous piecewise differential system, and this is not possible because system (8) has a unique center, and if there exist 2 limit cicles of the continuous piecewise differential system (8)+(10) they must surround the same equilibrium point. Of course those computations do not prove that in the cases 2.1.1.1 and 3.1.1.1 cannot exist two limit cicles, but they provide numerical evidences that probably in these cases the upper bound that we have found is two, but the maximum number of limit cicles can be one.

Subcase 3.1.1.2: We consider v_7 of (51). As in **subcase 3.1.1.1.** the continuous piecewise differential systems formed by (8) and (10) can have at most two limit cycle in this case. Indeed solving the algebraic system (12), we obtain three pairs (y_1,y_2). The first pair is

$$y_1=y_2=-\frac{8c_2}{3b_1}.$$

The second pair is

$$y_1=-\frac{1}{4}\sqrt{2}\sqrt{\mathcal{R}_2}-\frac{3\sqrt{\mathcal{R}_1}+\beta_1b_1^2(64\sqrt{2}c_2+21)-2\beta_1^2b_1(16c_2+3\sqrt{2})+3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1-\beta_1)},$$

$$y_2=\frac{\sqrt{\mathcal{R}_2}}{2\sqrt{2}}-\frac{3\sqrt{\mathcal{R}_1}+\beta_1b_1^2(64\sqrt{2}c_2+21)-2\beta_1^2b_1(16c_2+3\sqrt{2})+3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1-\beta_1)}.$$

And the third pair is

$$y_1=\frac{-3\sqrt{2}\beta_1b_1^2(2\sqrt{2}b_1-\beta_1)\sqrt{\mathcal{R}_3}-21\beta_1b_1^2+6\sqrt{2}\beta_1^2b_1+3\sqrt{\mathcal{R}_1}-64\sqrt{2}\beta_1b_1^2c_2+32\beta_1^2b_1c_2-3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1-\beta_1)},$$

$$y_2=\frac{3\sqrt{2}\beta_1b_1^2\sqrt{\mathcal{R}_3}(2\sqrt{2}b_1-\beta_1)-21\beta_1b_1^2+6\sqrt{2}\beta_1^2b_1+3\sqrt{\mathcal{R}_1}-64\sqrt{2}\beta_1b_1^2c_2+32\beta_1^2b_1c_2-3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1-\beta_1)},$$

with

$$\mathcal{R}_1=b_1^2(14\sqrt{2}\beta_1b_1^3-39\beta_1^2b_1^2+16\sqrt{2}\beta_1^3b_1+2b_1^4-4\beta_1^4),$$

$$\mathcal{R}_2=-\frac{12\sqrt{2}\beta_1b_1^4-31\beta_1^2b_1^3+\sqrt{2}b_1^2(3\sqrt{\mathcal{R}_1}+10\beta_1^3)-2\beta_1^4b_1-9\beta_1b_1\sqrt{\mathcal{R}_1}+2\sqrt{2}\beta_1^2\sqrt{\mathcal{R}_1}+6b_1^5}{b_1^3\beta_1^2(\beta_1-2\sqrt{2}b_1)^2},$$

and

$$\mathcal{R}_3=\frac{-12\sqrt{2}\beta_1b_1^4+31\beta_1^2b_1^3+\sqrt{2}b_1^2(3\sqrt{\mathcal{R}_1}-10\beta_1^3)+2\beta_1^4b_1-9\beta_1b_1\sqrt{\mathcal{R}_1}+2\sqrt{2}\beta_1^2\sqrt{\mathcal{R}_1}-6b_1^5}{b_1^3\beta_1^2(\beta_1-2\sqrt{2}b_1)^2}.$$

The invariant straight lines for the generalized systems (8) and (10) are given respectively by

$$y=-\frac{8c_2}{3b_1}-\frac{\alpha_1x+1}{\beta_1},$$

which intersects the $y-$axis at the point $y_{01}=-\frac{8c_2}{3b_1}-\frac{1}{\beta_1}$, and

$$y=-\frac{x\left(2a_1b_1-\sqrt{2}a_1\beta_1+2\alpha_1\beta_1+\sqrt{2}\alpha_1b_1\right)}{2\left(b_1^2+\beta_1^2\right)}-\frac{16c_2+3\sqrt{2}}{6b_1},$$

which intersects the $y-$axis at the point $y_{02} = -\frac{16c_2+3\sqrt{2}}{6b_1}$. The third pair (y_1, y_2) give $y_1 < y_2$, and in addition we have

- If $y_{01} < y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the third pair (y_1, y_2)
- If $y_{01} > y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the secondd pair (y_1, y_2);
- If $y_{01} = y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the second and the third pair of $y_1,\ y_2$.

Then the continuous piecewise differential systems formed by (8) and (10) can have at most one limit cycle

Subcase 3.1.1.3: We consider v_9 of (51). As in **subcase 3.1.1.1.** the continuous piecewise differential systems formed by (8) and (10) can have at most two limit cycle in this case. Solving the algebraic system (12), we obtain three pairs (y_1, y_2). The first pair is

$$y_1 = y_2 = -\frac{8c_2}{3b_1}.$$

The second pair is

$$y_1 = -\frac{-21\beta_1 b_1^2 + 3\sqrt{2}\beta_1 b_1^2\sqrt{\mathcal{S}_2}(2\sqrt{2}b_1+\beta_1) - 6\sqrt{2}\beta_1^2 b_1 + 3\sqrt{\mathcal{S}_4} + 64\sqrt{2}\beta_1 b_1^2 c_2 + 32\beta_1^2 b_1 c_2 + 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)},$$

$$y_2 = \frac{21\beta_1 b_1^2 + 3\sqrt{2}\beta_1 b_1^2\sqrt{\mathcal{S}_2}(2\sqrt{2}b_1+\beta_1) + 6\sqrt{2}\beta_1^2 b_1 - 3\sqrt{\mathcal{S}_4} - 64\sqrt{2}\beta_1 b_1^2 c_2 - 32\beta_1^2 b_1 c_2 - 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)}.$$

And the third pair is

$$y_1 = \frac{-3\sqrt{2}\beta_1 b_1^2(2\sqrt{2}b_1+\beta_1)\sqrt{\mathcal{S}_3} + 21\beta_1 b_1^2 + 6\sqrt{2}\beta_1^2 b_1 + 3\sqrt{\mathcal{S}_4} - 64\sqrt{2}\beta_1 b_1^2 c_2 - 32\beta_1^2 b_1 c_2 - 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)},$$

$$y_2 = \frac{21\beta_1 b_1^2 + 3\sqrt{2}\beta_1 b_1^2\sqrt{\mathcal{S}_3}(2\sqrt{2}b_1+\beta_1) + 6\sqrt{2}\beta_1^2 b_1 + 3\sqrt{\mathcal{S}_4} - 64\sqrt{2}\beta_1 b_1^2 c_2 - 32\beta_1^2 b_1 c_2 - 3\sqrt{2}b_1^3}{12b_1^2\beta_1(2\sqrt{2}b_1+\beta_1)}.$$

With

$$\mathcal{S}_1 = -b_1^2(14\sqrt{2}\beta_1 b_1^3 + 39\beta_1^2 b_1^2 + 16\sqrt{2}\beta_1^3 b_1 - 2b_1^4 + 4\beta_1^4),$$

$$\mathcal{S}_2 = \frac{12\sqrt{2}\beta_1 b_1^4 + 31\beta_1^2 b_1^3 + \sqrt{2}b_1^2(10\beta_1^3 - 3\sqrt{\mathcal{S}_1}) + b_1(2\beta_1^4 - 9\beta_1\sqrt{\mathcal{S}_1}) - 2\sqrt{2}\beta_1^2\sqrt{\mathcal{S}_1} - 6b_1^5}{b_1^3\beta_1^2(2\sqrt{2}b_1+\beta_1)^2},$$

$$\mathcal{S}_3 = \frac{12\sqrt{2}\beta_1 b_1^4 + 31\beta_1^2 b_1^3 + \sqrt{2}b_1^2(3\sqrt{\mathcal{S}_1} + 10\beta_1^3) + b_1(9\beta_1\sqrt{\mathcal{S}_1} + 2\beta_1^4) + 2\sqrt{2}\beta_1^2\sqrt{\mathcal{S}_1} - 6b_1^5}{b_1^3\beta_1^2(2\sqrt{2}b_1+\beta_1)^2},$$

and

$$\mathcal{S}_4 = b_1^2(-14\sqrt{2}\beta_1 b_1^3 - 39\beta_1^2 b_1^2 - 16\sqrt{2}\beta_1^3 b_1 + 2b_1^4 - 4\beta_1^4).$$

Since the invariant straight lines for the generalized systems (8) and (10) are given respectively as follows

$$y = -\frac{8c_2}{3b_1} - \frac{\alpha_1 x + 1}{\beta_1},$$

which intersects the $y-$axis at the point $y_{01} = -\frac{8c_2}{3b_1} - \frac{1}{\beta_1}$, and

$$y = \frac{1}{6}\left(\frac{3\sqrt{2} - 16c_2}{b_1} - \frac{3x\left(2a_1 b_1 + \sqrt{2}a_1\beta_1 + 2\alpha_1\beta_1 - \sqrt{2}\alpha_1 b_1\right)}{b_1^2+\beta_1^2}\right),$$

which intersects the $y-$axis at the point $y_{02} = \frac{3\sqrt{2}-16c_2}{6b_1}$. In both pairs of $y_1,\ y_2$ give $y_1 < y_2$, and in addition we have

- If $y_{01} < y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the second pair (y_1, y_2);
- If $y_{01} > y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the third pair (y_1, y_2);
- If $y_{01} = y_{02}$ we get $y_{02} \in [y_1, y_2]$ of the second and the third pair of $y_1,\ y_2$.

Then the continuous piecewise differential systems formed by (8) and (10) can have at most one limit cycle.

Subcase 3.1.2: We consider z_9 of (50). The fourth equation of (34) becomes

$$9\sqrt{2}\left(\alpha_1 b_1 - a_1\beta_1\right)\left(b_1^2\left(6a_1\beta_1 - 32a_2\beta_1\right) + \left(3a_1 - 32a_2\right)\beta_1^3 + 9\alpha_1\beta_1^2 b_1 + 6\alpha_1 b_1^3\right) = 0.$$

All sets of real solution for this last equation is given as follows

$$\begin{array}{rcl} e_1 &=& \{\alpha_1 = \frac{32a_2\beta_1\left(b_1^2+\beta_1^2\right)-3a_1\left(2b_1^2\beta_1+\beta_1^3\right)}{9\beta_1^2 b_1+6b_1^3}, \\ e_2 &=& \{\beta_1 = \frac{\alpha_1 b_1}{a_1}\}, \\ e_3 &=& \{a_1 = 0, b_1 = 0\}, \\ e_4 &=& \{a_1 = 0, \alpha_1 = 0\}, \\ e_5 &=& \{a_2 = \frac{3a_1}{32}, b_1 = 0\} \quad \text{and} \\ e_6 &=& \{b_1 = 0, \beta_1 = 0\}. \end{array} \tag{57} \quad \texttt{\{e9\}}$$

The only allowed solution is e_1. Then we have

$$\alpha_1 = \frac{32a_2\beta_1\left(b_1^2+\beta_1^2\right) - 3a_1\left(2b_1^2\beta_1 + \beta_1^3\right)}{9\beta_1^2 b_1 + 6b_1^3}.$$

In this case the algebraic system (12) has no real solutions

$$y_1 = \frac{-\sqrt{2}b_1\left(8\gamma_2+3\right) + (3+3i)\beta_1}{6b_1^2} \quad \text{and} \quad y_2 = -\frac{\sqrt{2}b_1\left(8\gamma_2+3\right) + (-3+3i)\beta_1}{6b_1^2}$$

and then the continuous piecewise differential systems formed by (8) and (10) has no limit cycle in this case.

Subcase 3.2: We consider u_3 of (49). Solving the fifth equation of (34) we obtain all sets of real solutions as follows

$$\begin{array}{rcl} w_1 &=& \{b_1 = 0\}, \\ w_2 &=& \{\alpha_1 = 0\}, \\ w_3 &=& \{\alpha_2 = -\frac{8a_2\left(-32(8\gamma_2+3)c_2+128\sqrt{2}c_2^2+\sqrt{2}\left(64\gamma_2^2+48\gamma_2+27\right)\right)-27a_1\left(-32(8\gamma_2+3)c_2+128\sqrt{2}c_2^2+\sqrt{2}(8\gamma_2+3)^2\right)}{16\left(-64\sqrt{2}(8\gamma_2+3)c_2+512c_2^2+256\gamma_2^2+192\gamma_2+27\right)}, \\ w_4 &=& \{a_2 = \frac{3a_1}{8}, c_2 = \frac{16\gamma_2+3}{16\sqrt{2}}\} \quad \text{and} \\ w_5 &=& \{a_2 = \frac{3a_1}{8}, c_2 = \frac{16\gamma_2+9}{16\sqrt{2}}\}. \end{array} \tag{58} \quad \texttt{\{w9\}}$$

The allowed solutions are w_3, w_4 and w_5. Then we have three different subcases

Subcase 3.2.1: We consider w_3 of (58). We solve now the fourth equation of (34) and we obtain

$$\begin{array}{rcl} e_1 &=& \{b_1 = 0\}, \\ e_2 &=& \{\alpha_1 = 0\}, \\ e_3 &=& \{\alpha_1 = [(3a_1 - 8a_2)(-8192(8\gamma_2+3)c_2^3 + 192\sqrt{2}(256\gamma_2^2 + 192\gamma_2 + 87)c_2^2 - 16(2048\gamma_2^3 \\ &+& 2304\gamma_2^2 + 2088\gamma_2 + 567)c_2 + 16384\sqrt{2}c_2^4 + \sqrt{2}(4096\gamma_2^4 + 6144\gamma_2^3 + 8352\gamma_2^2 + 4536\gamma_2 + 729))] \\ &/& [54(1536(8\gamma_2+3)c_2^2 - 24\sqrt{2}(256\gamma_2^2 + 192\gamma_2 + 33)c_2 - 4096\sqrt{2}c_2^3 + 2048\gamma_2^3 + 2304\gamma_2^2 + 792\gamma_2 + 81)]\}, \\ e_4 &=& \{\gamma_2 = \sqrt{2}c_2 - \frac{3}{8}\}, \\ e_5 &=& \{a_2 = \frac{3a_1}{8}, \gamma_2 = \sqrt{2}c_2 - \frac{9}{16}\} \quad \text{and} \\ e_6 &=& \{a_2 = \frac{3a_1}{8}, \gamma_2 = \sqrt{2}c_2 - \frac{3}{16}\}. \end{array}$$

The allowed solution is e_3. The algebraic system (12) in this case has complex solutions and then the continuous piecewise differential systems formed by (8) and (10) cannot have limit cycles.

Subcase 3.2.2: We consider w_4 of (58). We solve now the fourth equation of (34) which becomes

$$-27\alpha_1 b_1\left(3\sqrt{2}a_1 + 6\alpha_1 - 8\alpha_2\right) = 0,$$

we obtain

$$\begin{array}{rcl} e_1 &=& \{b_1 = 0\}, \\ e_2 &=& \{\alpha_1 = 0\} \quad \text{and} \\ e_3 &=& \{\alpha_2 = \frac{3}{8}\left(\sqrt{2}a_1 + 2\alpha_1\right)\}. \end{array}$$

The allowed solution is e_3. The algebraic system (12) in this case gives $y_1 = y_2 = -\frac{4\sqrt{2}\gamma_2}{3b_1}$ or has complex solutions

$$y_1 = -\frac{i\left(3\sqrt{10} - 8i\sqrt{2}\gamma_2\right)}{6b_1} \quad \text{and} \quad y_2 = \frac{i\left(3\sqrt{10} + 8i\sqrt{2}\gamma_2\right)}{6b_1},$$

and then the continuous piecewise differential systems formed by (8) and (10) cannot have limit cycles.

Subcase 3.2.3: We consider w_5 of (58). We solve now the fourth equation of (34) which becomes

$$27\alpha_1 b_1 \left(-3\sqrt{2}a_1 + 6\alpha_1 + 8\alpha_2\right) = 0,$$

we obtain

$$\begin{aligned} e_1 &= \{b_1 = 0\}, \\ e_2 &= \{\alpha_1 = 0\} \quad \text{and} \\ e_3 &= \{\alpha_2 = \tfrac{3}{8}\left(\sqrt{2}a_1 - 2\alpha_1\right)\}. \end{aligned}$$

The allowed solution is e_3. The algebraic system (12) in this case gives $y_1 = y_2 = \frac{-4\sqrt{2}\gamma_2 - 3\sqrt{2}}{3b_1}$, or has complex solutions

$$y_1 = -\frac{i\left(-8i\sqrt{2}\gamma_2 - 6i\sqrt{2} + 3\sqrt{10}\right)}{6b_1} \quad \text{and} \quad y_2 = \frac{-8\sqrt{2}\gamma 2 - 6\sqrt{2} + 3i\sqrt{10}}{6\text{b}1},$$

and then the continuous piecewise differential systems formed by (8) and (10) cannot have limit cycles.

Case 4: We consider s_8 of (35). We solve now the remaining equations of (34) which are equivalent to the system

$$\begin{aligned} &a_1 b_2 \left(8\gamma_2 + 3\right)\left(b_2\left(c_1^2 - \gamma_1\right) + \beta_1 c_2\right) = 0, \\ &a_1\left(8\gamma_2 + 3\right)\left(a_1 b_2\left(3b_2 c_1\left(\gamma_1 + 1\right) + \beta_1\gamma_2\left(4\gamma_2 + 3\right) - 16\beta_1 c_2^2\right) - 3\left(a_2\beta_1^2 c_2 + \alpha_1 b_2^2\left(c_1^2 - \gamma_1\right)\right)\right) = 0, \\ &a_1 b_2\left(8\gamma_2 + 3\right)\left(a_1 b_2\left(3c_1 - 32c_2\right) - 3a_2\beta_1 + 3\alpha_1 b_2\right) = 0, \\ &16a_1 b_2^3\left(8\gamma_2 + 3\right) = 0. \end{aligned}$$

we get one of the following sets of real solutions

$$\begin{aligned} s_1 &= \{a_1 = 0\}, \\ s_2 &= \{\gamma_2 = -\tfrac{3}{8}\}, \\ s_3 &= \{a_2 = 0, b_2 = 0\}, \\ s_4 &= \{b_2 = 0, c_2 = 0\} \quad \text{and} \\ s_5 &= \{b_2 = 0, \beta_1 = 0\}. \end{aligned}$$

No allowed solution exists. Then no continuous piecewise differential systems formed by (8) and (10).

Case 5: We consider s_{10} of (35). We solve now the sixth equation of (34) which is equivalent to $-8b_2^2\left(\sqrt{2}a_2 + \alpha_2\right) = 0$, and we obtain one of the following sets of real solutions

$$\begin{aligned} s_1 &= \{b_2 = 0\} \quad \text{and} \\ s_2 &= \{\alpha_2 = -\sqrt{2}a_2\}. \end{aligned}$$

No solution is allowed, and hence no continuous piecewise differential systems formed by (8) and (10).

Case 6: We consider s_{12} of (35). Solving the four remaing equations of (34), and we obtain one of the following sets of real solutions

$$\begin{aligned} &\left\{a_1 = 0, a_2 = \tfrac{\alpha_2}{\sqrt{2}}\right\}, \\ &\left\{a_2 = \tfrac{\alpha_2}{\sqrt{2}}, b_2 = 0\right\}, \\ &\left\{b_2 = 0, \beta_1 = 0\right\}, \\ &\left\{a_2 = \tfrac{\alpha_2}{\sqrt{2}}, \beta_1 = 0\right\} \quad \text{and} \\ &\left\{a_2 = 0, b_2 = 0, \alpha_2 = 0\right\}. \end{aligned}$$

No solution is allowed, and hence no continuous piecewise differential systems formed by (8) and (10).

∎

3. Conclusion

In order to complete the remaining statements for Theorem 1, we have considered the planar continuous piecewise differential systems formed by two a quadratic isochronous centers separated by the $y-$axis, having in $x > 0$ the quadratic isochronous center (8), and when $x < 0$ one of the systems (8), (9), (10) or (10). These four continuous piecewise differential systems have two as an upper bound of the number of crossing limit cycles, and there are a realized examples having one crossing limit cycle. See Theorem 1. In conclusion for these four different continuous piecewise differential systems, we have proved the extension of the 16th Hilbert problem to them.

4. Appendix 1

{app1}

The coefficients G_1, G_2 and G_3 in Subcase 2.1.1.1 in the proof of statement (g) of Theorem 1 are given as follows

$$G_1 = b_1^3(59049(4503599627370496c_2^{14}+3940649673949184\sqrt{2}(8\gamma_2+3)c_2^{13}+35184372088832(1528\gamma_2(4\gamma_2+3)+873)c_2^{12}+8796093022208\sqrt{2}(8\gamma_2+3)(1672\gamma_2(4\gamma_2+3)+981)c_2^{11}+1099511627776(\gamma_2(4\gamma_2+3)(83072\gamma_2(4\gamma_2+3)+97101)+28350)c_2^{10}+137438953472\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(95744\gamma_2(4\gamma_2+3)+114057)+33939)c_2^9+805306368(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(111584\gamma_2(4\gamma_2+3)+198423)+1880577)+11874195)c_2^8+402653184\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(18656\gamma_2(4\gamma_2+3)+33669)+323919)+2076435)c_2^7+176160768(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(10912\gamma_2(4\gamma_2+3)+20007)+195615)+637389)c_2^6+1048576\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(89056\gamma_2(4\gamma_2+3)+166077)+1652157)+5479893)c_2^5+32768(8\gamma_2+3)^4(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(103312\gamma_2(4\gamma_2+3)+196209)+994113)+13442031)c_2^4+8192\sqrt{2}(8\gamma_2+3)^5(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(10832\gamma_2(4\gamma_2+3)+20979)+54189)+1495179)c_2^3+512(8\gamma_2+3)^6(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(24832\gamma_2(4\gamma_2+3)+49113)+517833)+911979)c_2^2+64\sqrt{2}(8\gamma_2+3)^7(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(2176\gamma_2(4\gamma_2+3)+4401)+47385)+85293)c_2+(8\gamma_2+3)^8(32\gamma_2(4\gamma_2+3)+27)(256\gamma_2(4\gamma_2+3)(22\gamma_2(4\gamma_2+3)+27)+2187))c_1^6-52488(63050394783186944c_2^{15}+53620983063379968\sqrt{2}(8\gamma_2+3)c_2^{14}+1583296743997440(448\gamma_2(4\gamma_2+3)+257)c_2^{13}+1099511627776\sqrt{2}(8\gamma_2+3)(170672\gamma_2(4\gamma_2+3)+101367)c_2^{12}+1649267441664(4\gamma_2(4\gamma_2+3)(170176\gamma_2(4\gamma_2+3)+201501)+238221)c_2^{11}+19327352832\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+3)(62568\gamma_2(4\gamma_2+3)+76261)+2968965)c_2^{10}+4294967296(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(1864192\gamma_2(4\gamma_2+3)+3402369)+66117303)+26720037)c_2^9+2415919104\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(320\gamma_2(4\gamma_2+3)(51568\gamma_2(4\gamma_2+3)+97083)+19447857)+4048380)c_2^8+226492416(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(39776\gamma_2(4\gamma_2+3)+100493)+1517967)+10156995)+50813001)c_2^7+262144\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(358204\gamma_2(4\gamma_2+3)+951381)+120327525)+209920653)+2184071607)c_2^6+196608(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(53152\gamma_2(4\gamma_2+3)+154773)+10561995)+78664203)+216637659)c_2^5+36864\sqrt{2}(8\gamma_2+3)^3(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4036\gamma_2(4\gamma_2+3)+15251)+2456757)+20609559)+31121739)c_2^4-512(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(20\gamma_2(4\gamma_2+3)(3392\gamma_2(4\gamma_2+3)-8631)-500823)-22910283)-82570185)c_2^3-24\sqrt{2}(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(1964\gamma_2(4\gamma_2+3)+2817)+116721)-1677429)-19860147)c_2^2-36(8\gamma_2+3)^6(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(768\gamma_2(4\gamma_2+3)+1447)+26487)+44469)-137781)c_2-\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^7(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(1504\gamma_2(4\gamma_2+3)+1863)+9477))c_1^5+4374(3891110078048108544c_2^{16}+3211066534315163648\sqrt{2}(8\gamma_2+3)c_2^{15}+1688849860263936(24320\gamma_2(4\gamma_2+3)+14091)c_2^{14}+316659348799488\sqrt{2}(8\gamma_2+3)(33040\gamma_2(4\gamma_2+3)+20249)c_2^{13}+2199023255552(512\gamma_2(4\gamma_2+3)(53185\gamma_2(4\gamma_2+3)+65277)+10204623)c_2^{12}+206158430208\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(592304\gamma_2(4\gamma_2+3)+773493)+16001199)c_2^{11}+77309411328(256\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(607552\gamma_2(4\gamma_2+3)+1211461)+1588041)+87802137)c_2^{10}+8589934592\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(3061520\gamma_2(4\gamma_2+3)+6789231)+309823137)+71758386)c_2^9+1207959552(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(529760\gamma_2(4\gamma_2+3)+1723731)+31276287)+240995979)+674964333)c_2^8+226492416\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(20680\gamma_2(4\gamma_2+3)+96791)+8365815)+17964909)+217464831)c_2^7+2097152(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(46816\gamma_2(4\gamma_2+3)+1316943)+12078153)+2755209573)+8807046813)+21181880133)c_2^6-393216\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(90352\gamma_2(4\gamma_2+3)-273921)-32568183)-66914181)-3691662561)-4672803249)c_2^5-147456(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(2770\gamma_2(4\gamma_2+3)-3289)-4904829)-87842313)-634426101)-830747259)c_2^4-256\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(15860\gamma_2(4\gamma_2+3)-28989)-35660169)-633167847)-4615420743)-12271149837)c_2^3-96(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(4160\gamma_2(4\gamma_2+3)-38703)-953991)-62963001)-225271935)-1198753749)c_2^2+288\sqrt{2}(8\gamma_2+3)^5(\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(176\gamma_2(4\gamma_2+3)+1847)+266697)+2012769)+27903933)+4605822)c_2+(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(1472\gamma_2(4\gamma_2+3)(8\gamma_2^2+6\gamma_2+9)+18225)+164025)+531441))c_1^4-1296(35740566642812256256c_2^{17}+28570836036038426624\sqrt{2}(8\gamma_2+3)c_2^{16}+2251799813685248(156496\gamma_2(4\gamma_2+3)+92349)c_2^{15}+70368744177664\sqrt{2}(8\gamma_2+3)(1222960\gamma_2(4\gamma_2+3)+794349)c_2^{14}+35184372088832(8\gamma_2(4\gamma_2+3)(1662640\gamma_2(4\gamma_2+3)+2186937)+5656959)c_2^{13}+549755813888\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(1624276\gamma_2(4\gamma_2+3)+2428641)+55619379)c_2^{12}+34359738368(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(9093536\gamma_2(4\gamma_2+3)+21828825)+520907193)+1981526247)c_2^{11}+1073741824\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$$

$3)(8729776\gamma_2(4\gamma_2+3)+27712233)+1538398251)+6476856633)c_2^{10}+268435456(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(4922720\gamma_2(4\gamma_2+3)+30752613)+722209851)+6445072233)+39815710623)c_2^9+8388608\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(128\gamma_2(4\gamma_2+3)(152680\gamma_2(4\gamma_2+3)+11672919)+2756410479)+27369608805)+90671150115)c_2^8-2097152(64\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(503624\gamma_2(4\gamma_2+3)-3493251)-688213503)-10972936431)-37593631143)-$
$380949604335)c_2^7-65536\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(1663984\gamma_2(4\gamma_2+3)-7804791)-$
$882556155)-14978195109)-107632778535)-568326843585)c_2^6-16384(32\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(124856\gamma_2(4\gamma_2+3)-950589)-32569047)-2979587025)-16289473653)-348760104867)-1473838416567)c_2^5+$
$512\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(5735\gamma_2(4\gamma_2+3)+107514)+191405025)+$
$4420770453)+12615799167)+566991745695)+1255878519237)c_2^4+256(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(122080\gamma_2(4\gamma_2+3)+455139)+39605355)+61267347)+778442967)+77082859845)+92650892499)c_2^3+$
$8\sqrt{2}(8\gamma_2+3)^3(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(18184\gamma_2(4\gamma_2+3)+8397)-510057)+$
$51484167)+674752923)+23163918867)+33705582543)c_2^2+2(8\gamma_2+3)^4(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(3232\gamma_2(4\gamma_2+3)-23679)-1084023)-7433613)-125951517)+43046721)+1420541793)c_2-\sqrt{2}\gamma_2(4\gamma_2+$
$3)(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(416\gamma_2(4\gamma_2+3)+8721)+238383)+570807)+$
$3720087))c_1^3+9(7821419487252849885184c_2^{18}+6045920370158305542144\sqrt{2}(8\gamma_2+3)c_2^{17}+864691128455135232(82928\gamma_2(4\gamma_2+3)+$
$50403)c_2^{16}+27021597764222976\sqrt{2}(8\gamma_2+3)(618544\gamma_2(4\gamma_2+3)+440529)c_2^{15}+13510798882111488(4\gamma_2(4\gamma_2+3)(1586320\gamma_2(4\gamma_2+$
$3)+2336103)+3277179)c_2^{14}+211106232532992\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(715316\gamma_2(4\gamma_2+3)+1328763)+34660737)c_2^{13}+$
$39582418599936(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(1178888\gamma_2(4\gamma_2+3)+3863173)+27006867)+454358565)c_2^{12}+412316860416\sqrt{2}(8\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(665252\gamma_2(4\gamma_2+3)+3892149)+1057756563)+4992478497)c_2^{11}+51539607552(512\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(56\gamma_2(4\gamma_2+3)(313984\gamma_2(4\gamma_2+3)+9974967)+1028916351)+644559201)+68959891323)c_2^{10}-1073741824\sqrt{2}(8\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(6971360\gamma_2(4\gamma_2+3)-89007381)-6526225485)-73004332293)-264178894635)c_2^9-$
$201326592(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(5293024\gamma_2(4\gamma_2+3)-37997325)-2230608699)-80476063245)-$
$152131081419)-1672746272145)c_2^8-25165824\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(489776\gamma_2(4\gamma_2+$
$3)-4812207)-711677367)-14737165839)-122830333389)-727740287271)c_2^7+6291456(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(286496\gamma_2(4\gamma_2+3)+2406567)+43182909)+2860677891)+39373245531)+1978479398061)+$
$2353970611251)c_2^6+196608\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(206512\gamma_2(4\gamma_2+$
$3)-579483)+79925697)+4748690691)+39533298939)+1099633802355)+2798449794657)c_2^5+12288(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(124240\gamma_2(4\gamma_2+3)-1558569)-18976923)+3132532683)+47747692809)+$
$1106031289113)+5920657166601)+24501375705501)c_2^4-9216\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(358\gamma_2(4\gamma_2+3)+17059)+9683307)-301807539)-5539520097)-134870218911)-742484192715)-$
$784612583667)c_2^3-1536(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(37888\gamma_2(4\gamma_2+$
$3)+417195)+4524579)-322339743)-4945963923)-7236632097)-78732452709)-166203389781)c_2^2-192\sqrt{2}(8\gamma_2+3)^3(2\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(3808\gamma_2(4\gamma_2+3)+20067)-869697)-44761329)-$
$2073859929)-11120402925)-58242213513)-15109399071)c_2-(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(4\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(248\gamma_2(4\gamma_2+3)-1053)-742851)-5885217)-19663317)-243931419)-1162261467))c_1^2-$
$24(2361183241434822606848c_2^{19}+1761664059039262179328\sqrt{2}(8\gamma_2+3)c_2^{18}+6917529027641081856(2896\gamma_2(4\gamma_2+3)+1839)c_2^{17}+$
$972777519512027136\sqrt{2}(8\gamma_2+3)(4560\gamma_2(4\gamma_2+3)+3703)c_2^{16}+81064793292668928(8\gamma_2(4\gamma_2+3)(32816\gamma_2(4\gamma_2+3)+56963)+$
$176373)c_2^{15}+105553116266496\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(644456\gamma_2(4\gamma_2+3)+1621131)+24413913)c_2^{14}+131941395333120(256\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(134176\gamma_2(4\gamma_2+3)+692919)+714663)+53030619)c_2^{13}+824633720832\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(619184\gamma_2(4\gamma_2+3)+9820401)+203726853)+1080500715)c_2^{12}-103079215104(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(428032\gamma_2(4\gamma_2+3)-10945131)-402746013)-4583251431)-16786701225)c_2^{11}-536870912\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(1280\gamma_2(4\gamma_2+3)(133364\gamma_2(4\gamma_2+3)-1377279)-5492737251)-36416968281)-295893771363)c_2^{10}-536870912(4\gamma_2(4\gamma_2+$
$3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(1512896\gamma_2(4\gamma_2+3)-14556051)-2830869891)-15952623165)-553059750879)-$
$417619151433)c_2^9+25165824\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(70064\gamma_2(4\gamma_2+3)+721365)+$
$1335076965)+18911826621)+366397455051)+594764458695)c_2^8+37748736(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(39872\gamma_2(4\gamma_2+3)-261411)+15486417)+756152577)+47558442951)+320706318627)+399559605273)c_2^7+$
$294912\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16296\gamma_2(4\gamma_2+3)-287443)+65935737)+$
$1944233847)+16216223733)+454429891251)+2335727304603)c_2^6-49152(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+$
$3)(2\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(586\gamma_2(4\gamma_2+3)+24033)-51587199)-467180379)-82693752687)-867239849367)-8907538402305)-$
$9049152548457)c_2^5-3072\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(25712\gamma_2(4\gamma_2+$
$3)+46845)-63747243)-845437095)-2203435305)-731126235663)-7550590965129)-3884579149761)c_2^4-768(8\gamma_2+$
$3)^2(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(4864\gamma_2(4\gamma_2+3)-121263)-3478869)-$
$549289593)-10551682323)-107299532223)-278373578769)-581776434315)c_2^3+24\sqrt{2}(8\gamma_2+3)^3(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(1520\gamma_2(4\gamma_2+3)+58353)+14301441)+434161053)+3795256377)+$

$18986792607)+12526595811)+215793212373)c_2^2+16(8\gamma_2+3)^4(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(1984\gamma_2(4\gamma_2+3)+32355)+2565351)+13443489)+199998963)+1018772397)+11751754833)+3486784401)c_2+$
$\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(352\gamma_2(4\gamma_2+3)-$
$135)-3645)-19683))c_1+64(295147905179352825856c_2^{20}+212137556847659843584\sqrt{2}(8\gamma_2+3)c_2^{19}+57646075230342348800(40\gamma_2(4\gamma_2+$
$3)+27)c_2^{18}+324259173170675712\sqrt{2}(8\gamma_2+3)(1488\gamma_2(4\gamma_2+3)+1435)c_2^{17}+3377699720527872(32\gamma_2(4\gamma_2+3)(19832\gamma_2(4\gamma_2+3)+$
$42855)+591327)c_2^{16}+316659348799488\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(19160\gamma_2(4\gamma_2+3)+70893)+1249263)c_2^{15}+13194139533312(32\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(91904\gamma_2(4\gamma_2+3)+890301)+8779455)+90548361)c_2^{14}-2473901162496\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(720\gamma_2(4\gamma_2+3)-428701)-11373093)-69244551)c_2^{13}-51539607552(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(11474\gamma_2(4\gamma_2+$
$3)-169119)-136525257)-909075879)-7356711519)c_2^{12}-536870912\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(265424\gamma_2(4\gamma_2+3)-2876715)-1023792291)-8229027087)-74254320891)c_2^{11}-67108864(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(71104\gamma_2(4\gamma_2+3)-1284057)-275672079)-30805712397)-147688077195)-949653593883)c_2^{10}+8388608\sqrt{2}(8\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(118960\gamma_2(4\gamma_2+3)-1110987)+920049759)+15602051439)+$
$326967992685)+558593560719)c_2^9+18874368(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(18400\gamma_2(4\gamma_2+$
$3)-312821)+33449643)+1759740957)+29141622639)+205215463197)+265387168395)c_2^8-98304\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(340\gamma_2(4\gamma_2+3)+57027)-57169611)-1636300035)-29161648269)-$
$436621985685)-2378454806709)c_2^7-331776(128\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(20\gamma_2(4\gamma_2+3)-29)-97911)-57047463)-369417105)-277475625)-25428395529)-450976758219)c_2^6-3072\sqrt{2}(8\gamma_2+$
$3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(2512\gamma_2(4\gamma_2+3)-75729)+301401)-$
$43466139)-4711338189)-152635110561)-2059034673717)-1265415759423)c_2^5+1152(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(76\gamma_2(4\gamma_2+3)+2827)-2456271)-80394525)-120535047)+741950685)+$
$39013615251)+120286887381)c_2^4+24\sqrt{2}(8\gamma_2+3)^3(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+$
$3)(280\gamma_2(4\gamma_2+3)+369)-10363059)-352188891)-2035937349)-3714595443)+4424246325)+62891259381)c_2^3+(8\gamma_2+$
$3)^4(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)-23823)-338013)-$
$345692529)-1062232461)-1353580227)-3486784401)+15109399071)c_2^2-\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^5(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)+81)+729)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(112\gamma_2(4\gamma_2+3)+2529)+245673)+925101)+1948617)c_2-4\gamma_2^2(4\gamma_2+$
$3)^2(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)^2)),$

$G_2=-6b_1^4(531441(8\gamma_2+3)(8796093022208\sqrt{2}c_2^{12}+13194139533312(8\gamma_2+3)c_2^{11}+103079215104\sqrt{2}(704\gamma_2(4\gamma_2+3)+399)c_2^{10}+$
$42949672960(8\gamma_2+3)(704\gamma_2(4\gamma_2+3)+405)c_2^9+12079595520\sqrt{2}(8\gamma_2+3)^2(352\gamma_2(4\gamma_2+3)+207)c_2^8+2415919104(8\gamma_2+3)^3(352\gamma_2(4\gamma_2+$
$3)+213)c_2^7+88080384\sqrt{2}(8\gamma_2+3)^4(704\gamma_2(4\gamma_2+3)+441)c_2^6+9437184(8\gamma_2+3)^5(704\gamma_2(4\gamma_2+3)+459)c_2^5+11796480\sqrt{2}(8\gamma_2+$
$3)^6(22\gamma_2(4\gamma_2+3)+15)c_2^4+163840(8\gamma_2+3)^7(88\gamma_2(4\gamma_2+3)+63)c_2^3+384\sqrt{2}(8\gamma_2+3)^8(704\gamma_2(4\gamma_2+3)+531)c_2^2+96(8\gamma_2+$
$3)^9(64\gamma_2(4\gamma_2+3)+51)c_2+\sqrt{2}(8\gamma_2+3)^{10}(32\gamma_2(4\gamma_2+3)+27))c_1^6-78732(-140737488355328c_2^{14}+615726511554560\sqrt{2}(8\gamma_2+$
$3)c_2^{13}+981863883603968(8\gamma_2+3)^2c_2^{12}+137438953472\sqrt{2}(8\gamma_2+3)(39136\gamma_2(4\gamma_2+3)+22311)c_2^{11}+2147483648(128\gamma_2(4\gamma_2+$
$3)(127160\gamma_2(4\gamma_2+3)+146079)+5367951)c_2^{10}+268435456\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(274384\gamma_2(4\gamma_2+3)+322587)+6060015)c_2^9+$
$201326592(4\gamma_2(4\gamma_2+3)(320\gamma_2(4\gamma_2+3)(55088\gamma_2(4\gamma_2+3)+98037)+18575649)+14640021)c_2^8+100663296\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(73568\gamma_2(4\gamma_2+3)+135621)+2654775)+2156625)c_2^7+5505024(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(31504\gamma_2(4\gamma_2+$
$3)+61371)+1261413)+4281903)c_2^6+32768\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(164560\gamma_2(4\gamma_2+3)+353673)+7871499)+$
$28554201)c_2^5+8192(8\gamma_2+3)^4(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(89584\gamma_2(4\gamma_2+3)+245133)+6374133)+6461127)c_2^4-512\sqrt{2}(8\gamma_2+$
$3)^5(16\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)-3699)-389529)-1976319)c_2^3-8(8\gamma_2+3)^6(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(10672\gamma_2(4\gamma_2+3)+7209)-280179)-2932767)c_2^2-\sqrt{2}(8\gamma_2+3)^7(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(872\gamma_2(4\gamma_2+3)+1053)+$
$26973)-124659)c_2-\gamma_2(4\gamma_2+3)(8\gamma_2+3)^8(32\gamma_2(4\gamma_2+3)+27)(800\gamma_2(4\gamma_2+3)+459))c_1^5+4374(-24769797950537728c_2^{15}+$
$46443371157258240\sqrt{2}(8\gamma_2+3)c_2^{14}+52776558133248(24080\gamma_2(4\gamma_2+3)+13533)c_2^{13}+2199023255552\sqrt{2}(8\gamma_2+3)(195760\gamma_2(4\gamma_2+$
$3)+113283)c_2^{12}+103079215104(1280\gamma_2(4\gamma_2+3)(20428\gamma_2(4\gamma_2+3)+24147)+9113391)c_2^{11}+12884901888\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(829664\gamma_2(4\gamma_2+3)+1048173)+10486611)c_2^{10}+2147483648(10\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(682352\gamma_2(4\gamma_2+3)+1354977)+$
$56302533)+120045159)c_2^9+603979776\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(346720\gamma_2(4\gamma_2+3)+809037)+9406611)+$
$34930035)c_2^8+37748736(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(23870\gamma_2(4\gamma_2+3)+92931)+29075517)+57924153)+$
$657490203)c_2^7+1048576\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)(28336\gamma_2(4\gamma_2+3)+351171)+90000639)+$
$409445037)+1261923057)c_2^6-196608(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(7381\gamma_2(4\gamma_2+3)-28071)-34174791)-$
$354478437)-586966743)c_2^5-12288\sqrt{2}(8\gamma_2+3)^3(32\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(10384\gamma_2(4\gamma_2+3)-15549)-$
$838269)-47273463)-327934089)c_2^4-128(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(55880\gamma_2(4\gamma_2+3)-101187)-$
$41737437)-238169403)-1680672321)c_2^3-48\sqrt{2}(8\gamma_2+3)^5(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4960\gamma_2(4\gamma_2+3)-$
$28917)-2209923)-6042681)-84853413)c_2^2+48(8\gamma_2+3)^6(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(2560\gamma_2(4\gamma_2+3)(14\gamma_2(4\gamma_2+3)+195)+$
$908091)+9310059)+2007666)c_2+\sqrt{2}(8\gamma_2+3)^7(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(136\gamma_2(4\gamma_2+3)+567)+$
$2187)+19683))c_1^4-5832(-72057594037927936c_2^{16}+75435293758455808\sqrt{2}(8\gamma_2+3)c_2^{15}+211106232532992(10640\gamma_2(4\gamma_2+3)+$
$5949)c_2^{14}+9851624184872960\sqrt{2}(8\gamma_2+3)(76\gamma_2(4\gamma_2+3)+45)c_2^{13}+5497558138880(16\gamma_2(4\gamma_2+3)(50824\gamma_2(4\gamma_2+3)+63351)+$

$$312903)c_2^{12} + 51539607552\sqrt{2}(8\gamma_2 + 3)(64\gamma_2(4\gamma_2 + 3)(158896\gamma_2(4\gamma_2 + 3) + 231651) + 5147469)c_2^{11} + 1073741824(16\gamma_2(4\gamma_2 +$$
$$3)(64\gamma_2(4\gamma_2+3)(1143472\gamma_2(4\gamma_2+3)+2892825)+140222259)+533154879)c_2^{10}+134217728\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$$
$$3)(399520\gamma_2(4\gamma_2+3)+1644417)+48560877)+414357039)c_2^{9}+75497472(16\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(1024\gamma_2(4\gamma_2+3)(209\gamma_2(4\gamma_2+$$
$$3)+4266)+7305471)+338267907)+1053453843)c_2^{8}-2097152\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(40\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(13816\gamma_2(4\gamma_2+$$
$$3) - 106101) - 14430069) - 367467759) - 2430686475)c_2^{7} - 65536(224\gamma_2(4\gamma_2 + 3)(16\gamma_2(4\gamma_2 + 3)(16\gamma_2(4\gamma_2 + 3)(32\gamma_2(4\gamma_2 +$$
$$3)(39952\gamma_2(4\gamma_2 + 3) - 166383) - 18179397) - 293994765) - 2008656711) - 70545308859)c_2^{6} - 24576\sqrt{2}(8\gamma_2 + 3)(64\gamma_2(4\gamma_2 +$$
$$3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8224\gamma_2(4\gamma_2+3)-55431)-2926773)-195330447)-692743185)-7211949615)c_2^{5}+$$
$$512(8\gamma_2+3)^2(256\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(11120\gamma_2(4\gamma_2+3)+301887)+27684909)+237446235)+$$
$$442270449) + 19361281365)c_2^{4} + 64\sqrt{2}(8\gamma_2 + 3)^3(16\gamma_2(4\gamma_2 + 3)(16\gamma_2(4\gamma_2 + 3)(80\gamma_2(4\gamma_2 + 3)(128\gamma_2(4\gamma_2 + 3)(1112\gamma_2(4\gamma_2 + 3) +$$
$$4095)+1280043)+119921229)+991609857)+2966680809)c_2^{3}+24(8\gamma_2+3)^4(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$$
$$3)(496\gamma_2(4\gamma_2+3)+315)+36693)+8085339)+99536931)+185118615)c_2^{2}+\sqrt{2}(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$$
$$3)(32\gamma_2(4\gamma_2+3)(416\gamma_2(4\gamma_2+3)-1791)-122229)-1791153)+767637)+23914845)c_2-\gamma_2(4\gamma_2+3)(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)+$$
$$27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)+1431)+12879)+59049))c_1^3+27(-29975959119778021376c_2^{17}+19167320014088830976\sqrt{2}(8\gamma_2+$$
$$3)c_2^{16} + 4503599627370496(140912\gamma_2(4\gamma_2 + 3) + 77589)c_2^{15} + 562949953421312\sqrt{2}(8\gamma_2 + 3)(368000\gamma_2(4\gamma_2 + 3) + 223659)c_2^{14} +$$
$$35184372088832(320\gamma_2(4\gamma_2+3)(103684\gamma_2(4\gamma_2+3)+139959)+14693157)c_2^{13}+35184372088832\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(878716\gamma_2(4\gamma_2+$$
$$3) + 1601523) + 2521935)c_2^{12} + 68719476736(16\gamma_2(4\gamma_2 + 3)(64\gamma_2(4\gamma_2 + 3)(3467696\gamma_2(4\gamma_2 + 3) + 13236129) + 762612975) +$$
$$3207437433)c_2^{11}+549755813888\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(2628032\gamma_2(4\gamma_2+3)+33238593)+151400421)+45039807)c_2^{10}-$$
$$536870912(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(1183600\gamma_2(4\gamma_2+3)-16309341)-2169499383)-22400960589)-75380975811)c_2^{9}-$$
$$134217728\sqrt{2}(8\gamma_2 + 3)(8\gamma_2(4\gamma_2 + 3)(64\gamma_2(4\gamma_2 + 3)(20\gamma_2(4\gamma_2 + 3)(743776\gamma_2(4\gamma_2 + 3) - 5008437) - 259275897) - 12023978013) -$$
$$22000273029)c_2^{8}-4194304(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(844976\gamma_2(4\gamma_2+3)-6479235)-881545599)-$$
$$17001899109)-133103892735)-744344826705)c_2^{7}+1048576\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+$$
$$3)(155456\gamma_2(4\gamma_2+3)+1761759)+26852067)+5215346919)+46528965189)+141939328995)c_2^{6}+32768(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+$$
$$3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(163448\gamma_2(4\gamma_2+3)-396495)+105558957)+6094259937)+6109485102)+1305941324319)+$$
$$3187241401437)c_2^{5}+8192\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(112400\gamma_2(4\gamma_2+3)-$$
$$1293327)-948915)+512247159)+10635689367)+313465690881)+404175229407)c_2^{4}-512(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+$$
$$3)(\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(640\gamma_2(4\gamma_2+3)(1112\gamma_2(4\gamma_2+3)+69525)+32785317)-659801133)-989306946)-243884120751)-$$
$$322520382639)c_2^{3}-64\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(33344\gamma_2(4\gamma_2+3)+$$
$$370791) + 2622213) - 221125383) - 4764722859) - 17912218905) - 47050066053)c_2^{2} - 32(8\gamma_2 + 3)^4(2\gamma_2(4\gamma_2 + 3)(128\gamma_2(4\gamma_2 +$$
$$3)(\gamma_2(4\gamma_2 + 3)(16\gamma_2(4\gamma_2 + 3)(64\gamma_2(4\gamma_2 + 3)(3728\gamma_2(4\gamma_2 + 3) + 20385) - 1188999) - 112160295) - 122231430) - 6882692391) -$$
$$2195382771)c_2 - \sqrt{2}(8\gamma_2 + 3)^5(32\gamma_2(4\gamma_2 + 3) + 27)(16\gamma_2(4\gamma_2 + 3)(8\gamma_2(4\gamma_2 + 3)(16\gamma_2(4\gamma_2 + 3)(64\gamma_2(4\gamma_2 + 3)(26\gamma_2(4\gamma_2 + 3) -$$
$$27) - 43011) - 1082565) - 4782969) - 14348907))c_1^2 - 12(-64563604257983430656c_2^{18} + 25940733853654056960\sqrt{2}(8\gamma_2 + 3)c_2^{17} +$$
$$5188146770730811392(190\gamma_2(4\gamma_2+3)+101)c_2^{16}+3377699720527872\sqrt{2}(8\gamma_2+3)(92848\gamma_2(4\gamma_2+3)+57621)c_2^{15}+316659348799488(160\gamma_2(4\gamma_2+$$
$$3)(32536\gamma_2(4\gamma_2+3)+48819)+2740311)c_2^{14}+39582418599936\sqrt{2}(8\gamma_2+3)(544\gamma_2(4\gamma_2+3)(7112\gamma_2(4\gamma_2+3)+17575)+4256721)c_2^{13}+$$
$$824633720832(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(1038064\gamma_2(4\gamma_2+3)+7470357)+126380763)+579754989)c_2^{12}-154618822656\sqrt{2}(8\gamma_2+$$
$$3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(35072\gamma_2(4\gamma_2+3)-1683393)-74672793)-397657593)c_2^{11}-4831838208(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$$
$$3)(256\gamma_2(4\gamma_2+3)(91300\gamma_2(4\gamma_2+3)-808369)-549450027)-3264549723)-24286529295)c_2^{10}-67108864\sqrt{2}(8\gamma_2+3)(256\gamma_2(4\gamma_2+$$
$$3)(\gamma_2(4\gamma_2+3)(320\gamma_2(4\gamma_2+3)(239888\gamma_2(4\gamma_2+3)-1900737)-2573984061)-2317559526)-155008434213)c_2^{9}+75497472(8\gamma_2(4\gamma_2+$$
$$3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(5456\gamma_2(4\gamma_2+3)+405189)+496835775)+6550265475)+119844675981)+184552669701)c_2^{8}+$$
$$9437184\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(3658\gamma_2(4\gamma_2+3)-22987)+19991403)+1378335123)+$$
$$3481961337)+90973454751)c_2^{7}+98304(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(84728\gamma_2(4\gamma_2+3)-$$
$$1259427)+249961491)+7318577043)+117711257481)+1582571087703)+7791361373061)c_2^{6}-36864\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+$$
$$3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(241\gamma_2(4\gamma_2+3)+12486)-27433917)-183363183)-11751942915)-$$
$$39893681547) - 796810217499)c_2^{5} - 9216(4\gamma_2(4\gamma_2 + 3)(32\gamma_2(4\gamma_2 + 3)(32\gamma_2(4\gamma_2 + 3)(128\gamma_2(4\gamma_2 + 3)(\gamma_2(4\gamma_2 + 3)(160\gamma_2(4\gamma_2 +$$
$$3)(2896\gamma_2(4\gamma_2+3)+5555)-33107157)-105492132)-16851439341)-334277156637)-3300035502159)-1622387867769)c_2^{4}-$$
$$384\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4672\gamma_2(4\gamma_2+3)-104547)-$$
$$23537709)-887214627)-8089785171)-155946224187)-191658350799)-758956737951)c_2^{3}+72(8\gamma_2+3)^2(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$$
$$3)(8\gamma_2(4\gamma_2 + 3)(8\gamma_2(4\gamma_2 + 3)(16\gamma_2(4\gamma_2 + 3)(64\gamma_2(4\gamma_2 + 3)(848\gamma_2(4\gamma_2 + 3) + 35427) + 8681067) + 253595043) + 2086955685) +$$
$$9755662437)+24034419225)+96725982087)c_2^{2}+9\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$$
$$3)(1024\gamma_2(4\gamma_2+3)(13\gamma_2(4\gamma_2+3)+218)+579717)+12313539)+21434787)+97253703)+1994498073)+4261625379)c_2+\gamma_2(4\gamma_2+$$
$$3)(8\gamma_2 + 3)^4(32\gamma_2(4\gamma_2 + 3) + 27)(16\gamma_2(4\gamma_2 + 3)(8\gamma_2(4\gamma_2 + 3) + 81) + 729)(16\gamma_2(4\gamma_2 + 3)(32\gamma_2(4\gamma_2 + 3) + 27)(184\gamma_2(4\gamma_2 + 3) -$$
$$135)-19683))c_1+32(-9223372036854775808c_2^{19}+2305843009213693952\sqrt{2}(8\gamma_2+3)c_2^{18}+108086391056891904(976\gamma_2(4\gamma_2+3)+$$
$$483)c_2^{17}+6755399441055744\sqrt{2}(8\gamma_2+3)(4816\gamma_2(4\gamma_2+3)+2997)c_2^{16}+105553116266496(32\gamma_2(4\gamma_2+3)(46568\gamma_2(4\gamma_2+3)+79563)+$$
$$947673)c_2^{15} + 13194139533312\sqrt{2}(8\gamma_2 + 3)(64\gamma_2(4\gamma_2 + 3)(14492\gamma_2(4\gamma_2 + 3) + 52887) + 1657449)c_2^{14} + 2473901162496(16\gamma_2(4\gamma_2 +$$
$$3)(16\gamma_2(4\gamma_2 + 3)(13328\gamma_2(4\gamma_2 + 3) + 296067) + 5726295) + 28169937)c_2^{13} - 1236950581248\sqrt{2}(8\gamma_2 + 3)(8\gamma_2(4\gamma_2 + 3)(8\gamma_2(4\gamma_2 +$$

$3)(14512\gamma_2(4\gamma_2+3)-203129)-2768895)-8254791)c_2^{12}-1610612736(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(61400\gamma_2(4\gamma_2+$
$3)-511953)-247821849)-1720462599)-14085493785)c_2^{11}-67108864\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(103136\gamma_2(4\gamma_2+3)-1037943)-423575973)-3754547559)-34924275171)c_2^{10}+8388608(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(64\gamma_2(4\gamma_2+3)(151184\gamma_2(4\gamma_2+3)-1193967)+755240679)+12633531363)+256609639521)+423514131201)c_2^{9}+12582912\sqrt{2}(8\gamma_2+$
$3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(44384\gamma_2(4\gamma_2+3)-637989)+22563819)+1890504765)+21223105083)+$
$18784569465)c_2^{8}+98304(32\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(376\gamma_2(4\gamma_2+3)-166491)+$
$42349635)+1469145681)+6786075105)+408571427457)+2209108652001)c_2^{7}-12288\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(6832\gamma_2(4\gamma_2+3)-11217)-4553847)-134931825)-12856946535)-112423646979)-$
$671532567687)c_2^{6}-3072(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4592\gamma_2(4\gamma_2+$
$3)-120471)+10676205)+57287007)-1120551003)-129612495951)-2026708180569)-1315665631737)c_2^{5}+3456\sqrt{2}(8\gamma_2+$
$3)^3(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)+3101)-234009)-2853927)-102296925)+$
$368249247)+2382450003)c_2^{4}+72(8\gamma_2+3)^2(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(336\gamma_2(4\gamma_2+3)+349)-6596181)-226708713)-1371063105)-3095525727)-74933181)+22685621967)c_2^{3}+3\sqrt{2}(8\gamma_2+$
$3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)-7305)-3400623)-$
$108267435)-82924479)-1713897225)-2625849981)+2711943423)c_2^{2}-\gamma_2(4\gamma_2+3)(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+$
$81)+729)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(208\gamma_2(4\gamma_2+3)+4869)+467289)+1712421)+3365793)c_2-4\sqrt{2}\gamma_2^2(4\gamma_2+$
$3)^2(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)^2))$, and
$G_3=9b_1^5(531441(8796093022208c_2^{12}+6597069766656\sqrt{2}(8\gamma_2+3)c_2^{11}+103079215104(704\gamma_2(4\gamma_2+3)+399)c_2^{10}+21474836480\sqrt{2}(8\gamma_2+$
$3)(704\gamma_2(4\gamma_2+3)+405)c_2^{9}+12079595520(8\gamma_2+3)^2(352\gamma_2(4\gamma_2+3)+207)c_2^{8}+1207959552\sqrt{2}(8\gamma_2+3)^3(352\gamma_2(4\gamma_2+3)+213)c_2^{7}+$
$88080384(8\gamma_2+3)^4(704\gamma_2(4\gamma_2+3)+441)c_2^{6}+4718592\sqrt{2}(8\gamma_2+3)^5(704\gamma_2(4\gamma_2+3)+459)c_2^{5}+11796480(8\gamma_2+3)^6(22\gamma_2(4\gamma_2+3)+$
$15)c_2^{4}+81920\sqrt{2}(8\gamma_2+3)^7(88\gamma_2(4\gamma_2+3)+63)c_2^{3}+384(8\gamma_2+3)^8(704\gamma_2(4\gamma_2+3)+531)c_2^{2}+48\sqrt{2}(8\gamma_2+3)^9(64\gamma_2(4\gamma_2+3)+51)c_2+$
$(8\gamma_2+3)^{10}(32\gamma_2(4\gamma_2+3)+27))c_1^6-944784(61572651155456c_2^{13}+45079976738816\sqrt{2}(8\gamma_2+3)c_2^{12}+103079215104(4672\gamma_2(4\gamma_2+3)+$
$2661)c_2^{11}+34359738368\sqrt{2}(8\gamma_2+3)(2816\gamma_2(4\gamma_2+3)+1647)c_2^{10}+268435456(80\gamma_2(4\gamma_2+3)(19360\gamma_2(4\gamma_2+3)+22707)+531927)c_2^{9}+$
$1207959552\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8096\gamma_2(4\gamma_2+3)+9813)+11853)c_2^{8}+12582912(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(26048\gamma_2(4\gamma_2+$
$3)+33129)+167265)c_2^{7}+352321536\sqrt{2}(8\gamma_2+3)^3(\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+963)+324)c_2^{6}+589824(8\gamma_2+3)^4(8\gamma_2(4\gamma_2+$
$3)(3344\gamma_2(4\gamma_2+3)+5169)+15363)c_2^{5}+4096\sqrt{2}(8\gamma_2+3)^5(80\gamma_2(4\gamma_2+3)(880\gamma_2(4\gamma_2+3)+1773)+62613)c_2^{4}+128(8\gamma_2+$
$3)^6(16\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+7407)+76707)c_2^{3}-384\sqrt{2}(8\gamma_2+3)^7(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)-3)-297)c_2^{2}-(8\gamma_2+$
$3)^8(16\gamma_2(4\gamma_2+3)(544\gamma_2(4\gamma_2+3)+333)-1215)c_2-2\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^9(32\gamma_2(4\gamma_2+3)+27))c_1^5+13122(22517998136852480c_2^{14}+$
$16008889300418560\sqrt{2}(8\gamma_2+3)c_2^{13}+2199023255552(74960\gamma_2(4\gamma_2+3)+43299)c_2^{12}+137438953472\sqrt{2}(8\gamma_2+3)(229760\gamma_2(4\gamma_2+$
$3)+141363)c_2^{11}+8589934592(16\gamma_2(4\gamma_2+3)(925760\gamma_2(4\gamma_2+3)+1159713)+5758533)c_2^{10}+42949672960\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(31768\gamma_2(4\gamma_2+3)+44433)+120447)c_2^{9}+4026531840(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(78496\gamma_2(4\gamma_2+3)+178281)+127332)+$
$1867941)c_2^{8}+50331648\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(56320\gamma_2(4\gamma_2+3)+175203)+1180737)+9630333)c_2^{7}+22020096(8\gamma_2+$
$3)^2(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(3520\gamma_2(4\gamma_2+3)+29241)+251019)+2312631)c_2^{6}-131072\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(25520\gamma_2(4\gamma_2+3)-139509)-3235545)-16707951)c_2^{5}-40960(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(1298\gamma_2(4\gamma_2+3)-$
$2781)-660231)-3692385)c_2^{4}-2560\sqrt{2}(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(3968\gamma_2(4\gamma_2+3)-8775)-135351)-1565163)c_2^{3}-$
$32(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(6080\gamma_2(4\gamma_2+3)-30753)-410427)-4721733)c_2^{2}+16\sqrt{2}(8\gamma_2+3)^7(32\gamma_2(4\gamma_2+$
$3)(4\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)+1809)+9963)+111537)c_2+(8\gamma_2+3)^8(8\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)+27)(40\gamma_2(4\gamma_2+$
$3)+27))c_1^4-69984(11258999068426240c_2^{15}+7740561859543040\sqrt{2}(8\gamma_2+3)c_2^{14}+21990232555520(3472\gamma_2(4\gamma_2+3)+2061)c_2^{13}+$
$549755813888\sqrt{2}(8\gamma_2+3)(25120\gamma_2(4\gamma_2+3)+17073)c_2^{12}+17179869184(176\gamma_2(4\gamma_2+3)(16960\gamma_2(4\gamma_2+3)+24399)+1476873)c_2^{11}+$
$2147483648\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(112640\gamma_2(4\gamma_2+3)+214281)+1386477)c_2^{10}+12884901888(5\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(10912\gamma_2(4\gamma_2+3)+41805)+301563)+396819)c_2^{9}+201326592\sqrt{2}(8\gamma_2+3)(20\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+$
$9423)+338985)+1988469)c_2^{8}-6291456(8\gamma_2+3)^2(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(7040\gamma_2(4\gamma_2+3)-71199)-784161)-8070273)c_2^{7}-$
$262144\sqrt{2}(8\gamma_2+3)^3(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(12320\gamma_2(4\gamma_2+3)-68877)-871641)-9692055)c_2^{6}-16384(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(4840\gamma_2(4\gamma_2+3)-36261)-1972917)-11844063)c_2^{5}+2048\sqrt{2}(8\gamma_2+3)^5(80\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)+2781)+79461)+2645541)c_2^{4}+64(8\gamma_2+3)^6(160\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+2457)+40095)+3243321)c_2^{3}+$
$8\sqrt{2}(8\gamma_2+3)^7(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(640\gamma_2(4\gamma_2+3)+81)+10449)+303993)c_2^{2}+4(8\gamma_2+3)^8(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(160\gamma_2(4\gamma_2+3)-621)-2673)+6561)c_2-27\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^9(32\gamma_2(4\gamma_2+3)+27))c_1^3+81(14411518807585587200c_2^{16}+$
$9547631210025451520\sqrt{2}(8\gamma_2+3)c_2^{15}+6755399441055744(13280\gamma_2(4\gamma_2+3)+8241)c_2^{14}+70368744177664\sqrt{2}(8\gamma_2+3)(215600\gamma_2(4\gamma_2+$
$3)+171459)c_2^{13}+10995116277760(32\gamma_2(4\gamma_2+3)(144632\gamma_2(4\gamma_2+3)+261405)+3275559)c_2^{12}+824633720832\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(242720\gamma_2(4\gamma_2+3)+755493)+5864859)c_2^{11}+8589934592(32\gamma_2(4\gamma_2+3)(2920\gamma_2(4\gamma_2+3)(3520\gamma_2(4\gamma_2+3)+36603)+118670589)+$
$1113354315)c_2^{10}-1073741824\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(344960\gamma_2(4\gamma_2+3)-6820533)-75439107)-793854027)c_2^{9}-$
$603979776(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(149600\gamma_2(4\gamma_2+3)-1142751)-44930457)-507514491)-1822681521)c_2^{8}-$
$2147483648\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(40\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(1078\gamma_2(4\gamma_2+3)-9333)-925587)-29953881)-29596671)c_2^{7}+$
$524288(128\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(24640\gamma_2(4\gamma_2+3)+940671)+21925485)+992114325)+$

$$
4274484939)+102025393641)c_2^6+196608\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(20\gamma_2(4\gamma_2+3)(3616\gamma_2(4\gamma_2+
3)-6009)+1133487)+39062007)+793170225)+10379849733)c_2^5+4096(8\gamma_2+3)^2(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+
3)(128\gamma_2(4\gamma_2+3)(2645\gamma_2(4\gamma_2+3)-26136)+1629477)+179334729)+4360250331)+30926854701)c_2^4-2048\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+
3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(800\gamma_2(4\gamma_2+3)+115497)+33939)-30192993)-409347351)-1516201173)c_2^3-
384(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+8331)-351)-11982573)-39503781)-
291761109)c_2^2-16\sqrt{2}(8\gamma_2+3)^5(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(1280\gamma_2(4\gamma_2+3)+7227)-15795)-
1948617)-22851963)-81310473)c_2-(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(320\gamma_2^2(4\gamma_2+3)^2-6561)-
59049)-531441))c_1^2-144(6341068275337658368c_2^{17}+4035225266123964416\sqrt{2}(8\gamma_2+3)c_2^{16}+4503599627370496(7976\gamma_2(4\gamma_2+3)+
5283)c_2^{15}+140737488355328\sqrt{2}(8\gamma_2+3)(39920\gamma_2(4\gamma_2+3)+39411)c_2^{14}+2199023255552(32\gamma_2(4\gamma_2+3)(235480\gamma_2(4\gamma_2+3)+589131)+
8438823)c_2^{13}+274877906944\sqrt{2}(8\gamma_2+3)(256\gamma_2(4\gamma_2+3)(11147\gamma_2(4\gamma_2+3)+69507)+10267641)c_2^{12}-8589934592(32\gamma_2(4\gamma_2+
3)(8\gamma_2(4\gamma_2+3)(29504\gamma_2(4\gamma_2+3)-6545907)-70427961)-737840583)c_2^{11}-2147483648\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+
3)(149248\gamma_2(4\gamma_2+3)-1696977)-24709455)-304505487)c_2^{10}-67108864(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(487520\gamma_2(4\gamma_2+
3)-4492647)-267871779)-3708547659)-15154801191)c_2^9-33554432\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+
3)(19360\gamma_2(4\gamma_2+3)-696699)-56124009)-1932030063)-2203465923)c_2^8+524288(512\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+
3)(2\gamma_2(4\gamma_2+3)(88000\gamma_2(4\gamma_2+3)-423261)+9452133)+1213136919)+1493145819)+153420226065)c_2^7+131072\sqrt{2}(8\gamma_2+
3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)(1232\gamma_2(4\gamma_2+3)-16839)+11239641)+430173423)+4499461629)+
29985731739)c_2^6-4096(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(40\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(496\gamma_2(4\gamma_2+3)+70011)-1175877)-
156493701)-19858237749)-266410133271)-656224941123)c_2^5-1024\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+
3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(5360\gamma_2(4\gamma_2+3)+22491)-6841665)-146805291)-4481641953)-59743357191)-73834692453)c_2^4-
128(8\gamma_2+3)^2(128\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(5440\gamma_2(4\gamma_2+3)-89019)-11718675)-
197419761)-2836576179)-1163324349)-22972600107)c_2^3+16\sqrt{2}(8\gamma_2+3)^3(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+
3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)+12555)+771525)+5181003)+69264477)+1784047437)+2195382771)c_2^2+(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+
3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(608\gamma_2(4\gamma_2+3)+10287)+406539)+518319)+24977727)+157837977)+
387420489)c_2+256\sqrt{2}\gamma_2^3(4\gamma_2+3)^3(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729))c_1+64(4611686018427387904c_2^{18}+
2810246167479189504\sqrt{2}(8\gamma_2+3)c_2^{17}+13510798882111488(1744\gamma_2(4\gamma_2+3)+1263)c_2^{16}+20266198323167232\sqrt{2}(8\gamma_2+3)(166\gamma_2(4\gamma_2+
3)+215)c_2^{15}+52776558133248(16\gamma_2(4\gamma_2+3)(9920\gamma_2(4\gamma_2+3)+37989)+308529)c_2^{14}+824633720832\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+
3)(8456\gamma_2(4\gamma_2+3)+151845)+3372381)c_2^{13}-51539607552(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(45616\gamma_2(4\gamma_2+3)-861873)-23305671)-
138661875)c_2^{12}-3221225472\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(73984\gamma_2(4\gamma_2+3)-809253)-18279837)-268027785)c_2^{11}-
201326592(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(54208\gamma_2(4\gamma_2+3)-704625)-102240387)-1730283687)-7827786945)c_2^{10}+
16777216\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(36080\gamma_2(4\gamma_2+3)-230949)+4718331)+391378959)+7796915253)c_2^9+
25165824(2\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(51040\gamma_2(4\gamma_2+3)-694773)+19038645)+2764517445)+
14576882067)+6169498569)c_2^8+196608\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(10\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+
3)-21633)+1508571)+122552919)+1397464569)+40192784415)c_2^7-36864(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+
3)(2\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(2240\gamma_2(4\gamma_2+3)+5573)-3293451)-23760945)-880465059)-53802187983)-147097377243)c_2^6-
49152\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(680\gamma_2(4\gamma_2+3)-14049)-399411)-
7680015)-1704147579)-3962955537)-3030808023)c_2^5+384(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+
3)(1792\gamma_2(4\gamma_2+3)(5\gamma_2(4\gamma_2+3)+276)-2337255)-70005141)-147515337)+19515753549)+47397628467)+130130237583)c_2^4+
48\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)+2589)-
1006749)-66568149)-179423667)-524886561)+2319739965)+11751754833)c_2^3+3(8\gamma_2+3)^2(256\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+
3)(16\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)-6183)-770391)-48208041)-70996581)-336402153)-
43046721)+1937102445)c_2^2-12\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+
3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+195)+4617)+32805)+59049)c_2-2\gamma_2^2(4\gamma_2+3)^2(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+
3)(8\gamma_2(4\gamma_2+3)+81)+729)^2)).
$$

5. Appendix 2

{app2}

The coefficients H_1, H_2 and H_3 in Subcase 3.1.1.1 in the proof of statement (g) of Theorem 1 are given as follows

$$
H_1=-b_1^3(-59049(-4503599627370496c_2^{14}+3940649673949184\sqrt{2}(8\gamma_2+3)c_2^{13}-35184372088832(1528\gamma_2(4\gamma_2+3)+873)c_2^{12}+
8796093022208\sqrt{2}(8\gamma_2+3)(1672\gamma_2(4\gamma_2+3)+981)c_2^{11}-1099511627776(\gamma_2(4\gamma_2+3)(83072\gamma_2(4\gamma_2+3)+97101)+28350)c_2^{10}+
137438953472\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(95744\gamma_2(4\gamma_2+3)+114057)+33939)c_2^9-805306368(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(111584\gamma_2(4\gamma_2+
3)+198423)+1880577)+11874195)c_2^8+402653184\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(18656\gamma_2(4\gamma_2+3)+33669)+323919)+
2076435)c_2^7-176160768(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(10912\gamma_2(4\gamma_2+3)+20007)+195615)+637389)c_2^6+1048576\sqrt{2}(8\gamma_2+
3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(89056\gamma_2(4\gamma_2+3)+166077)+1652157)+5479893)c_2^5-32768(8\gamma_2+3)^4(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+
$$

$$
3)(103312\gamma_2(4\gamma_2+3)+196209)+994113)+13442031)c_2^4+8192\sqrt{2}(8\gamma_2+3)^5(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(10832\gamma_2(4\gamma_2+3)+
20979)+54189)+1495179)c_2^3-512(8\gamma_2+3)^6(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(24832\gamma_2(4\gamma_2+3)+49113)+517833)+911979)c_2^2+
64\sqrt{2}(8\gamma_2+3)^7(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(2176\gamma_2(4\gamma_2+3)+4401)+47385)+85293)c_2-(8\gamma_2+3)^8(32\gamma_2(4\gamma_2+3)+27)(256\gamma_2(4\gamma_2+
3)(22\gamma_2(4\gamma_2+3)+27)+2187))c_1^6+52488(-63050394783186944c_2^{15}+53620983063379968\sqrt{2}(8\gamma_2+3)c_2^{14}-1583296743997440(448\gamma_2(4\gamma_2+
3)+257)c_2^{13}+1099511627776\sqrt{2}(8\gamma_2+3)(170672\gamma_2(4\gamma_2+3)+101367)c_2^{12}-1649267441664(4\gamma_2(4\gamma_2+3)(170176\gamma_2(4\gamma_2+3)+
201501)+238221)c_2^{11}+19327352832\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+3)(62568\gamma_2(4\gamma_2+3)+76261)+2968965)c_2^{10}-4294967296(2\gamma_2(4\gamma_2+
3)(32\gamma_2(4\gamma_2+3)(1864192\gamma_2(4\gamma_2+3)+3402369)+66117303)+26720037)c_2^9+2415919104\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(320\gamma_2(4\gamma_2+
3)(51568\gamma_2(4\gamma_2+3)+97083)+19447857)+4048380)c_2^8-226492416(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(39776\gamma_2(4\gamma_2+3)+
100493)+1517967)+10156995)+50813001)c_2^7+262144\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(358204\gamma_2(4\gamma_2+3)+
951381)+120327525)+209920653)+2184071607)c_2^6-196608(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(53152\gamma_2(4\gamma_2+
3)+154773)+10561995)+78664203)+216637659)c_2^5+36864\sqrt{2}(8\gamma_2+3)^3(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4036\gamma_2(4\gamma_2+
3)+15251)+2456757)+20609559)+31121739)c_2^4+512(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(20\gamma_2(4\gamma_2+3)(3392\gamma_2(4\gamma_2+
3)-8631)-500823)-22910283)-82570185)c_2^3-24\sqrt{2}(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(1964\gamma_2(4\gamma_2+
3)+2817)+116721)-1677429)-19860147)c_2^2+36(8\gamma_2+3)^6(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(768\gamma_2(4\gamma_2+3)+
1447)+26487)+44469)-137781)c_2-\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^7(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(1504\gamma_2(4\gamma_2+3)+1863)+
9477))c_1^5-4374(-38911100780481085 44c_2^{16}+3211066534315163648\sqrt{2}(8\gamma_2+3)c_2^{15}-1688849860263936(24320\gamma_2(4\gamma_2+3)+
14091)c_2^{14}+316659348799488\sqrt{2}(8\gamma_2+3)(33040\gamma_2(4\gamma_2+3)+20249)c_2^{13}-2199023255552(512\gamma_2(4\gamma_2+3)(53185\gamma_2(4\gamma_2+3)+65277)+
10204623)c_2^{12}+206158430208\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(592304\gamma_2(4\gamma_2+3)+773493)+16001199)c_2^{11}-77309411328(256\gamma_2(4\gamma_2+
3)(2\gamma_2(4\gamma_2+3)(607552\gamma_2(4\gamma_2+3)+1211461)+1588041)+87802137)c_2^{10}+8589934592\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+
3)(3061520\gamma_2(4\gamma_2+3)+6789231)+309823137)+71758386)c_2^9-1207959552(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(529760\gamma_2(4\gamma_2+
3)+1723731)+31276287)+240995979)+674964333)c_2^8+226492416\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+
3)(20680\gamma_2(4\gamma_2+3)+96791)+8365815)+17964909)+217464831)c_2^7-2097152(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+
3)(4\gamma_2(4\gamma_2+3)(46816\gamma_2(4\gamma_2+3)+1316943)+12078153)+2755209573)+8807046813)+21181880133)c_2^6-393216\sqrt{2}(8\gamma_2+
3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(90352\gamma_2(4\gamma_2+3)-273921)-32568183)-66914181)-3691662561)-
4672803249)c_2^5+147456(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(2770\gamma_2(4\gamma_2+3)-3289)-4904829)-
87842313)-634426101)-830747259)c_2^4-256\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(15860\gamma_2(4\gamma_2+
3)-28989)-35660169)-633167847)-4615420743)-12271149837)c_2^3+96(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+
3)(8\gamma_2(4\gamma_2+3)(4160\gamma_2(4\gamma_2+3)-38703)-953991)-62963001)-225271935)-1198753749)c_2^2+288\sqrt{2}(8\gamma_2+3)^5(\gamma_2(4\gamma_2+
3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(176\gamma_2(4\gamma_2+3)+1847)+266697)+2012769)+27903933)+4605822)c_2-(8\gamma_2+
3)^6(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(1472\gamma_2(4\gamma_2+3)(8\gamma_2^2+6\gamma_2+9)+18225)+164025)+531441))c_1^4+
1296(-3574056664281225 6256c_2^{17}+2857083603603842 6624\sqrt{2}(8\gamma_2+3)c_2^{16}-2251799813685248(156496\gamma_2(4\gamma_2+3)+92349)c_2^{15}+
70368744177664\sqrt{2}(8\gamma_2+3)(1222960\gamma_2(4\gamma_2+3)+794349)c_2^{14}-35184372088832(8\gamma_2(4\gamma_2+3)(1662640\gamma_2(4\gamma_2+3)+2186937)+
5656959)c_2^{13}+549755813888\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(1624276\gamma_2(4\gamma_2+3)+2428641)+55619379)c_2^{12}-34359738368(16\gamma_2(4\gamma_2+
3)(32\gamma_2(4\gamma_2+3)(9093536\gamma_2(4\gamma_2+3)+21828825)+520907193)+1981526247)c_2^{11}+1073741824\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+
3)(8729776\gamma_2(4\gamma_2+3)+27712233)+1538398251)+6476856633)c_2^{10}-268435456(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+
3)(4922720\gamma_2(4\gamma_2+3)+30752613)+722209851)+6445072233)+39815710623)c_2^9+8388608\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+
3)(128\gamma_2(4\gamma_2+3)(152680\gamma_2(4\gamma_2+3)+11672919)+2756410479)+27369608805)+90671150115)c_2^8+2097152(64\gamma_2(4\gamma_2+
3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(503624\gamma_2(4\gamma_2+3)-3493251)-688213503)-10972936431)-37593631143)-
380949604335)c_2^7-65536\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(1663984\gamma_2(4\gamma_2+3)-7804791)-
882556155)-14978195109)-107632778535)-568326843585)c_2^6+16384(32\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+
3)(8\gamma_2(4\gamma_2+3)(124856\gamma_2(4\gamma_2+3)-950589)-32569047)-2979587025)-16289473653)-348760104867)-1473838416567)c_2^5+
512\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(5735\gamma_2(4\gamma_2+3)+107514)+191405025)+
4420770453)+12615799167)+566991745695)+1255878519237)c_2^4-256(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+
3)(\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(122080\gamma_2(4\gamma_2+3)+455139)+39605355)+61267347)+778442967)+77082859845)+92650892499)c_2^3+
8\sqrt{2}(8\gamma_2+3)^3(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(18184\gamma_2(4\gamma_2+3)+8397)-510057)+
51484167)+674752923)+23163918867)+33705582543)c_2^2-2(8\gamma_2+3)^4(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+
3)(16\gamma_2(4\gamma_2+3)(3232\gamma_2(4\gamma_2+3)-23679)-1084023)-7433613)-125951517)+43046721)+1420541793)c_2-\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+
3)^5(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(416\gamma_2(4\gamma_2+3)+8721)+238383)+570807)+3720087))c_1^3-
9(-7821419487252849885184c_2^{18}+6045920370158305542144\sqrt{2}(8\gamma_2+3)c_2^{17}-864691128455135232(82928\gamma_2(4\gamma_2+3)+50403)c_2^{16}+
27021597764222976\sqrt{2}(8\gamma_2+3)(618544\gamma_2(4\gamma_2+3)+440529)c_2^{15}-13510798882111488(4\gamma_2(4\gamma_2+3)(1586320\gamma_2(4\gamma_2+3)+2336103)+
3277179)c_2^{14}+211106232532992\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(715316\gamma_2(4\gamma_2+3)+1328763)+34660737)c_2^{13}-39582418599936(64\gamma_2(4\gamma_2+
3)(8\gamma_2(4\gamma_2+3)(1178888\gamma_2(4\gamma_2+3)+3863173)+27006867)+454358565)c_2^{12}+412316860416\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+
3)(665252\gamma_2(4\gamma_2+3)+3892149)+1057756563)+4992478497)c_2^{11}-51539607552(512\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(56\gamma_2(4\gamma_2+3)(313984\gamma_2(4\gamma_2+
3)+9974967)+1028916351)+644559201)+68959891323)c_2^{10}-1073741824\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+
$$

$3)(6971360\gamma_2(4\gamma_2+3)-89007381)-6526225485)-73004332293)-264178894635)c_2^9+201326592(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(5293024\gamma_2(4\gamma_2+3)-37997325)-2230608699)-80476063245)-152131081419)-1672746272145)c_2^8-$
$25165824\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(489776\gamma_2(4\gamma_2+3)-4812207)-711677367)-$
$14737165839)-122830333389)-727740287271)c_2^7-6291456(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+$
$3)(286496\gamma_2(4\gamma_2+3)+2406567)+43182909)+2860677891)+39373245531)+1978479398061)+2353970611251)c_2^6+196608\sqrt{2}(8\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(206512\gamma_2(4\gamma_2+3)-579483)+79925697)+4748690691)+$
$39533298939)+1099633802355)+2798449794657)c_2^5-12288(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(124240\gamma_2(4\gamma_2+3)-1558569)-18976923)+3132532683)+47747692809)+1106031289113)+5920657166601)+$
$24501375705501)c_2^4-9216\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+$
$3)(358\gamma_2(4\gamma_2+3)+17059)+9683307)-301807539)-5539520097)-134870218911)-742484192715)-784612583667)c_2^3+$
$1536(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(37888\gamma_2(4\gamma_2+3)+417195)+$
$4524579)-322339743)-4945963923)-7236632097)-78732452709)-166203389781)c_2^2-192\sqrt{2}(8\gamma_2+3)^3(2\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(3808\gamma_2(4\gamma_2+3)+20067)-869697)-44761329)-2073859929)-$
$11120402925)-58242213513)-15109399071)c_2+(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+$
$3)(4\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(248\gamma_2(4\gamma_2+3)-1053)-742851)-5885217)-19663317)-243931419)-1162261467))c_1^2+$
$24(-2361183241434822606848c_2^{19}+1761664059039262179328\sqrt{2}(8\gamma_2+3)c_2^{18}-6917529027641081856(2896\gamma_2(4\gamma_2+3)+1839)c_2^{17}+$
$972777519512027136\sqrt{2}(8\gamma_2+3)(4560\gamma_2(4\gamma_2+3)+3703)c_2^{16}-81064793292668928(8\gamma_2(4\gamma_2+3)(32816\gamma_2(4\gamma_2+3)+56963)+$
$176373)c_2^{15}+105553116266496\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(644456\gamma_2(4\gamma_2+3)+1621131)+24413913)c_2^{14}-131941395333120(256\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(134176\gamma_2(4\gamma_2+3)+692919)+714663)+53030619)c_2^{13}+824633720832\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(619184\gamma_2(4\gamma_2+3)+9820401)+203726853)+1080500715)c_2^{12}+103079215104(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(428032\gamma_2(4\gamma_2+3)-10945131)-402746013)-4583251431)-16786701225)c_2^{11}-536870912\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(1280\gamma_2(4\gamma_2+3)(133364\gamma_2(4\gamma_2+3)-1377279)-5492737251)-36416968281)-295893771363)c_2^{10}+536870912(4\gamma_2(4\gamma_2+$
$3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(1512896\gamma_2(4\gamma_2+3)-14556051)-2830869891)-15952623165)-553059750879)-$
$417619151433)c_2^9+25165824\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(70064\gamma_2(4\gamma_2+3)+721365)+$
$1335076965)+18911826621)+366397455051)+594764458695)c_2^8-37748736(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(39872\gamma_2(4\gamma_2+3)-261411)+15486417)+756152577)+47558442951)+320706318627)+399559605273)c_2^7+$
$294912\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16296\gamma_2(4\gamma_2+3)-287443)+65935737)+$
$1944233847)+16216223733)+454429891251)+2335727304603)c_2^6+49152(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+$
$3)(2\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(586\gamma_2(4\gamma_2+3)+24033)-51587199)-467180379)-82693752687)-867239849367)-8907538402305)-$
$9049152548457)c_2^5-3072\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(25712\gamma_2(4\gamma_2+$
$3)+46845)-63747243)-845437095)-2203435305)-731126235663)-7550590965129)-3884579149761)c_2^4+768(8\gamma_2+$
$3)^2(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(4864\gamma_2(4\gamma_2+3)-121263)-3478869)-$
$549289593)-10551682323)-107299532223)-278373578769)-581776434315)c_2^3+24\sqrt{2}(8\gamma_2+3)^3(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(1520\gamma_2(4\gamma_2+3)+58353)+14301441)+434161053)+3795256377)+$
$18986792607)+12526595811)+215793212373)c_2^2-16(8\gamma_2+3)^4(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(1984\gamma_2(4\gamma_2+3)+32355)+2565351)+13443489)+199998963)+1018772397)+11751754833)+3486784401)c_2+$
$\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(352\gamma_2(4\gamma_2+3)-$
$135)-3645)-19683))c_1+64(295147905179352825856c_2^{20}-212137556847659843584\sqrt{2}(8\gamma_2+3)c_2^{19}+57646075230342348800(40\gamma_2(4\gamma_2+$
$3)+27)c_2^{18}-324259173170675712\sqrt{2}(8\gamma_2+3)(1488\gamma_2(4\gamma_2+3)+1435)c_2^{17}+3377699720527872(32\gamma_2(4\gamma_2+3)(19832\gamma_2(4\gamma_2+3)+$
$42855)+591327)c_2^{16}-316659348799488\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(19160\gamma_2(4\gamma_2+3)+70893)+1249263)c_2^{15}+13194139533312(32\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(91904\gamma_2(4\gamma_2+3)+890301)+8779455)+90548361)c_2^{14}+2473901162496\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(720\gamma_2(4\gamma_2+3)-428701)-11373093)-69244551)c_2^{13}-51539607552(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(11474\gamma_2(4\gamma_2+$
$3)-169119)-136525257)-909075879)-7356711519)c_2^{12}+536870912\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(265424\gamma_2(4\gamma_2+3)-2876715)-1023792291)-8229027087)-74254320891)c_2^{11}-67108864(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(71104\gamma_2(4\gamma_2+3)-1284057)-275672079)-30805712397)-147688077195)-949653593883)c_2^{10}-8388608\sqrt{2}(8\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(118960\gamma_2(4\gamma_2+3)-1110987)+920049759)+15602051439)+$
$326967992685)+558593560719)c_2^9+18874368(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(18400\gamma_2(4\gamma_2+$
$3)-312821)+33449643)+1759740957)+29141622639)+205215463197)+265387168395)c_2^8+98304\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(340\gamma_2(4\gamma_2+3)+57027)-57169611)-1636300035)-29161648269)-$
$436621985685)-2378454806709)c_2^7-331776(128\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(20\gamma_2(4\gamma_2+3)-29)-97911)-57047463)-369417105)-277475625)-25428395529)-450976758219)c_2^6+3072\sqrt{2}(8\gamma_2+$
$3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(2512\gamma_2(4\gamma_2+3)-75729)+301401)-$
$43466139)-4711338189)-152635110561)-2059034673717)-1265415759423)c_2^5+1152(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(76\gamma_2(4\gamma_2+3)+2827)-2456271)-80394525)-120535047)+741950685)+$

$39013615251)+120286887381)c_2^4-24\sqrt{2}(8\gamma_2+3)^3(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+$
$3)(280\gamma_2(4\gamma_2+3)+369)-10363059)-352188891)-2035937349)-3714595443)+4424246325)+62891259381)c_2^3+(8\gamma_2+$
$3)^4(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)-23823)-338013)-$
$345692529)-1062232461)-1353580227)-3486784401)+15109399071)c_2^2+\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^5(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)+81)+729)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(112\gamma_2(4\gamma_2+3)+2529)+245673)+925101)+1948617)c_2-4\gamma_2^2(4\gamma_2+$
$3)^2(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)^2)),$
$H_2=-6b_1^4(531441(8\gamma_2+3)(8796093022208\sqrt{2}c_2^{12}-13194139533312(8\gamma_2+3)c_2^{11}+103079215104\sqrt{2}(704\gamma_2(4\gamma_2+3)+399)c_2^{10}-$
$42949672960(8\gamma_2+3)(704\gamma_2(4\gamma_2+3)+405)c_2^9+12079595520\sqrt{2}(8\gamma_2+3)^2(352\gamma_2(4\gamma_2+3)+207)c_2^8-2415919104(8\gamma_2+3)^3(352\gamma_2(4\gamma_2+$
$3)+213)c_2^7+88080384\sqrt{2}(8\gamma_2+3)^4(704\gamma_2(4\gamma_2+3)+441)c_2^6-9437184(8\gamma_2+3)^5(704\gamma_2(4\gamma_2+3)+459)c_2^5+11796480\sqrt{2}(8\gamma_2+$
$3)^6(22\gamma_2(4\gamma_2+3)+15)c_2^4-163840(8\gamma_2+3)^7(88\gamma_2(4\gamma_2+3)+63)c_2^3+384\sqrt{2}(8\gamma_2+3)^8(704\gamma_2(4\gamma_2+3)+531)c_2^2-96(8\gamma_2+$
$3)^9(64\gamma_2(4\gamma_2+3)+51)c_2+\sqrt{2}(8\gamma_2+3)^{10}(32\gamma_2(4\gamma_2+3)+27))c_1^6-78732(140737488355328c_2^{14}+615726511554560\sqrt{2}(8\gamma_2+$
$3)c_2^{13}-981863883603968(8\gamma_2+3)^2c_2^{12}+137438953472\sqrt{2}(8\gamma_2+3)(39136\gamma_2(4\gamma_2+3)+22311)c_2^{11}-2147483648(128\gamma_2(4\gamma_2+$
$3)(127160\gamma_2(4\gamma_2+3)+146079)+5367951)c_2^{10}+268435456\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(274384\gamma_2(4\gamma_2+3)+322587)+6060015)c_2^9-$
$201326592(4\gamma_2(4\gamma_2+3)(320\gamma_2(4\gamma_2+3)(55088\gamma_2(4\gamma_2+3)+98037)+18575649)+14640021)c_2^8+100663296\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(73568\gamma_2(4\gamma_2+3)+135621)+2654775)+2156625)c_2^7-5505024(8\gamma_2+3)^2(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(31504\gamma_2(4\gamma_2+$
$3)+61371)+1261413)+4281903)c_2^6+32768\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(164560\gamma_2(4\gamma_2+3)+353673)+7871499)+$
$28554201)c_2^5-8192(8\gamma_2+3)^4(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(89584\gamma_2(4\gamma_2+3)+245133)+6374133)+6461127)c_2^4-512\sqrt{2}(8\gamma_2+$
$3)^5(16\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)-3699)-389529)-1976319)c_2^3+8(8\gamma_2+3)^6(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(10672\gamma_2(4\gamma_2+3)+7209)-280179)-2932767)c_2^2-\sqrt{2}(8\gamma_2+3)^7(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(872\gamma_2(4\gamma_2+3)+1053)+$
$26973)-124659)c_2+\gamma_2(4\gamma_2+3)(8\gamma_2+3)^8(32\gamma_2(4\gamma_2+3)+27)(800\gamma_2(4\gamma_2+3)+459))c_1^5+4374(24769797950537728c_2^{15}+$
$46443371157258240\sqrt{2}(8\gamma_2+3)c_2^{14}-52776558133248(24080\gamma_2(4\gamma_2+3)+13533)c_2^{13}+2199023255552\sqrt{2}(8\gamma_2+3)(195760\gamma_2(4\gamma_2+$
$3)+113283)c_2^{12}-103079215104(1280\gamma_2(4\gamma_2+3)(20428\gamma_2(4\gamma_2+3)+24147)+9113391)c_2^{11}+12884901888\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(829664\gamma_2(4\gamma_2+3)+1048173)+10486611)c_2^{10}-2147483648(10\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(682352\gamma_2(4\gamma_2+3)+1354977)+$
$56302533)+120045159)c_2^9+603979776\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(346720\gamma_2(4\gamma_2+3)+809037)+9406611)+$
$34930035)c_2^8-37748736(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)(23870\gamma_2(4\gamma_2+3)+92931)+29075517)+57924153)+$
$657490203)c_2^7+1048576\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)(28336\gamma_2(4\gamma_2+3)+351171)+90000639)+$
$409445037)+1261923057)c_2^6+196608(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(7381\gamma_2(4\gamma_2+3)-28071)-34174791)-$
$354478437)-586966743)c_2^5-12288\sqrt{2}(8\gamma_2+3)^3(32\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(10384\gamma_2(4\gamma_2+3)-15549)-$
$838269)-47273463)-327934089)c_2^4+128(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(55880\gamma_2(4\gamma_2+3)-101187)-$
$41737437)-238169403)-1680672321)c_2^3-48\sqrt{2}(8\gamma_2+3)^5(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4960\gamma_2(4\gamma_2+3)-$
$28917)-2209923)-6042681)-84853413)c_2^2-48(8\gamma_2+3)^6(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(2560\gamma_2(4\gamma_2+3)(14\gamma_2(4\gamma_2+3)+195)+$
$908091)+9310059)+2007666)c_2+\sqrt{2}(8\gamma_2+3)^7(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(136\gamma_2(4\gamma_2+3)+567)+$
$2187)+19683))c_1^4-5832(72057594037927936c_2^{16}+75435293758455808\sqrt{2}(8\gamma_2+3)c_2^{15}-211106232532992(10640\gamma_2(4\gamma_2+3)+$
$5949)c_2^{14}+9851624184872960\sqrt{2}(8\gamma_2+3)(76\gamma_2(4\gamma_2+3)+45)c_2^{13}-5497558138880(16\gamma_2(4\gamma_2+3)(50824\gamma_2(4\gamma_2+3)+63351)+$
$312903)c_2^{12}+51539607552\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(158896\gamma_2(4\gamma_2+3)+231651)+5147469)c_2^{11}-1073741824(16\gamma_2(4\gamma_2+$
$3)(64\gamma_2(4\gamma_2+3)(1143472\gamma_2(4\gamma_2+3)+2892825)+140222259)+533154879)c_2^{10}+134217728\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(399520\gamma_2(4\gamma_2+3)+1644417)+48560877)+414357039)c_2^9-75497472(16\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(1024\gamma_2(4\gamma_2+3)(209\gamma_2(4\gamma_2+$
$3)+4266)+7305471)+338267907)+1053453843)c_2^8-2097152\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(40\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(13816\gamma_2(4\gamma_2+$
$3)-106101)-14430069)-367467759)-2430686475)c_2^7+65536(224\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(39952\gamma_2(4\gamma_2+3)-166383)-18179397)-293994765)-2008656711)-70545308859)c_2^6-24576\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8224\gamma_2(4\gamma_2+3)-55431)-2926773)-195330447)-692743185)-7211949615)c_2^5-$
$512(8\gamma_2+3)^2(256\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(11120\gamma_2(4\gamma_2+3)+301887)+27684909)+237446235)+$
$442270449)+19361281365)c_2^4+64\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(1112\gamma_2(4\gamma_2+3)+$
$4095)+1280043)+119921229)+991609857)+2966680809)c_2^3-24(8\gamma_2+3)^4(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(496\gamma_2(4\gamma_2+3)+315)+36693)+8085339)+99536931)+185118615)c_2^2+\sqrt{2}(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(416\gamma_2(4\gamma_2+3)-1791)-122229)-1791153)+767637)+23914845)c_2+\gamma_2(4\gamma_2+3)(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)+$
$27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)+1431)+12879)+59049))c_1^3+27(29975959119778021376c_2^{17}+19167320014088830976\sqrt{2}(8\gamma_2+$
$3)c_2^{16}-4503599627370496(140912\gamma_2(4\gamma_2+3)+77589)c_2^{15}+562949953421312\sqrt{2}(8\gamma_2+3)(368000\gamma_2(4\gamma_2+3)+223659)c_2^{14}-$
$35184372088832(320\gamma_2(4\gamma_2+3)(103684\gamma_2(4\gamma_2+3)+139959)+14693157)c_2^{13}+35184372088832\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(878716\gamma_2(4\gamma_2+$
$3)+1601523)+2521935)c_2^{12}-68719476736(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(3467696\gamma_2(4\gamma_2+3)+13236129)+762612975)+$
$3207437433)c_2^{11}+549755813888\sqrt{2}(8\gamma_2+3)(\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(2628032\gamma_2(4\gamma_2+3)+33238593)+151400421)+45039807)c_2^{10}+$
$536870912(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(1183600\gamma_2(4\gamma_2+3)-16309341)-2169499383)-22400960589)-75380975811)c_2^9-$
$134217728\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(20\gamma_2(4\gamma_2+3)(743776\gamma_2(4\gamma_2+3)-5008437)-259275897)-12023978013)-$
$22000273029)c_2^8+4194304(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(844976\gamma_2(4\gamma_2+3)-6479235)-881545599)-$

$17001899109)-133103892735)-744344826705)c_2^7+1048576\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+$
$3)(155456\gamma_2(4\gamma_2+3)+1761759)+26852067)+5215346919)+46528965189)+141939328995)c_2^6-32768(16\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(163448\gamma_2(4\gamma_2+3)-396495)+105558957)+6094259937)+6109485102)+1305941324319)+$
$3187241401437)c_2^5+8192\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(112400\gamma_2(4\gamma_2+3)-$
$1293327)-948915)+512247159)+10635689367)+313465690881)+404175229407)c_2^4+512(8\gamma_2+3)^2(8\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(640\gamma_2(4\gamma_2+3)(1112\gamma_2(4\gamma_2+3)+69525)+32785317)-659801133)-989306946)-243884120751)-$
$322520382639)c_2^3-64\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(33344\gamma_2(4\gamma_2+3)+$
$370791)+2622213)-221125383)-4764722859)-17912218905)-47050066053)c_2^2+32(8\gamma_2+3)^4(2\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(3728\gamma_2(4\gamma_2+3)+20385)-1188999)-112160295)-122231430)-6882692391)-$
$2195382771)c_2-\sqrt{2}(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(26\gamma_2(4\gamma_2+3)-$
$27)-43011)-1082565)-4782969)-14348907))c_1^2-12(64563604257983430656c_2^{18}+25940733853654056960\sqrt{2}(8\gamma_2+3)c_2^{17}-$
$5188146770730811392(190\gamma_2(4\gamma_2+3)+101)c_2^{16}+3377699720527872\sqrt{2}(8\gamma_2+3)(92848\gamma_2(4\gamma_2+3)+57621)c_2^{15}-316659348799488(160\gamma_2(4\gamma_2+$
$3)(32536\gamma_2(4\gamma_2+3)+48819)+2740311)c_2^{14}+39582418599936\sqrt{2}(8\gamma_2+3)(544\gamma_2(4\gamma_2+3)(7112\gamma_2(4\gamma_2+3)+17575)+4256721)c_2^{13}-$
$824633720832(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(1038064\gamma_2(4\gamma_2+3)+7470357)+126380763)+579754989)c_2^{12}-154618822656\sqrt{2}(8\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(35072\gamma_2(4\gamma_2+3)-1683393)-74672793)-397657593)c_2^{11}+4831838208(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(256\gamma_2(4\gamma_2+3)(91300\gamma_2(4\gamma_2+3)-808369)-549450027)-3264549723)-24286529295)c_2^{10}-67108864\sqrt{2}(8\gamma_2+3)(256\gamma_2(4\gamma_2+$
$3)(\gamma_2(4\gamma_2+3)(320\gamma_2(4\gamma_2+3)(239888\gamma_2(4\gamma_2+3)-1900737)-2573984061)-2317559526)-155008434213)c_2^9-75497472(8\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(5456\gamma_2(4\gamma_2+3)+405189)+496835775)+6550265475)+119844675981)+184552669701)c_2^8+$
$9437184\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(3658\gamma_2(4\gamma_2+3)-22987)+19991403)+1378335123)+$
$3481961337)+90973454751)c_2^7-98304(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(84728\gamma_2(4\gamma_2+3)-$
$1259427)+249961491)+7318577043)+117711257481)+1582571087703)+7791361373061)c_2^6-36864\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)(241\gamma_2(4\gamma_2+3)+12486)-27433917)-183363183)-11751942915)-$
$39893681547)-796810217499)c_2^5+9216(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+$
$3)(2896\gamma_2(4\gamma_2+3)+5555)-33107157)-105492132)-16851439341)-334277156637)-3300035502159)-1622387867769)c_2^4-$
$384\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4672\gamma_2(4\gamma_2+3)-104547)-$
$23537709)-887214627)-8089785171)-155946224187)-191658350799)-758956737951)c_2^3-72(8\gamma_2+3)^2(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(848\gamma_2(4\gamma_2+3)+35427)+8681067)+253595043)+2086955685)+$
$9755662437)+24034419225)+96725982087)c_2^2+9\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(1024\gamma_2(4\gamma_2+3)(13\gamma_2(4\gamma_2+3)+218)+579717)+12313539)+21434787)+97253703)+1994498073)+4261625379)c_2-\gamma_2(4\gamma_2+$
$3)(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)+27)(184\gamma_2(4\gamma_2+3)-$
$135)-19683))c_1+32(9223372036854775808c_2^{19}+2305843009213693952\sqrt{2}(8\gamma_2+3)c_2^{18}-108086391056891904(976\gamma_2(4\gamma_2+3)+$
$483)c_2^{17}+6755399441055744\sqrt{2}(8\gamma_2+3)(4816\gamma_2(4\gamma_2+3)+2997)c_2^{16}-105553116266496(32\gamma_2(4\gamma_2+3)(46568\gamma_2(4\gamma_2+3)+79563)+$
$947673)c_2^{15}+13194139533312\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(14492\gamma_2(4\gamma_2+3)+52887)+1657449)c_2^{14}-2473901162496(16\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(13328\gamma_2(4\gamma_2+3)+296067)+5726295)+28169937)c_2^{13}-1236950581248\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(14512\gamma_2(4\gamma_2+3)-203129)-2768895)-8254791)c_2^{12}+1610612736(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(61400\gamma_2(4\gamma_2+$
$3)-511953)-247821849)-1720462599)-14085493785)c_2^{11}-67108864\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(103136\gamma_2(4\gamma_2+3)-1037943)-423575973)-3754547559)-34924275171)c_2^{10}-8388608(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(64\gamma_2(4\gamma_2+3)(151184\gamma_2(4\gamma_2+3)-1193967)+755240679)+12633531363)+256609639521)+423514131201)c_2^9+12582912\sqrt{2}(8\gamma_2+$
$3)(4\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(44384\gamma_2(4\gamma_2+3)-637989)+22563819)+1890504765)+21223105083)+$
$18784569465)c_2^8-98304(32\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(376\gamma_2(4\gamma_2+3)-166491)+$
$42349635)+1469145681)+6786075105)+408571427457)+2209108652001)c_2^7-12288\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(6832\gamma_2(4\gamma_2+3)-11217)-4553847)-134931825)-12856946535)-112423646979)-$
$671532567687)c_2^6+3072(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4592\gamma_2(4\gamma_2+$
$3)-120471)+10676205)+57287007)-1120551003)-129612495951)-2026708180569)-1315665631737)c_2^5+3456\sqrt{2}(8\gamma_2+$
$3)^3(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)+3101)-234009)-2853927)-102296925)+$
$368249247)+2382450003)c_2^4-72(8\gamma_2+3)^2(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(336\gamma_2(4\gamma_2+3)+349)-6596181)-226708713)-1371063105)-3095525727)-74933181)+22685621967)c_2^3+3\sqrt{2}(8\gamma_2+$
$3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)-7305)-3400623)-$
$108267435)-82924479)-1713897225)-2625849981)+2711943423)c_2^2+\gamma_2(4\gamma_2+3)(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+$
$81)+729)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(208\gamma_2(4\gamma_2+3)+4869)+467289)+1712421)+3365793)c_2-4\sqrt{2}\gamma_2^2(4\gamma_2+$
$3)^2(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)^2))$ and
$H_3=-9b_1^5(-531441(-8796093022208c_2^{12}+6597069766656\sqrt{2}(8\gamma_2+3)c_2^{11}-103079215104(704\gamma_2(4\gamma_2+3)+399)c_2^{10}+21474836480\sqrt{2}(8\gamma_2+$
$3)(704\gamma_2(4\gamma_2+3)+405)c_2^9-12079595520(8\gamma_2+3)^2(352\gamma_2(4\gamma_2+3)+207)c_2^8+1207959552\sqrt{2}(8\gamma_2+3)^3(352\gamma_2(4\gamma_2+3)+$
$213)c_2^7-88080384(8\gamma_2+3)^4(704\gamma_2(4\gamma_2+3)+441)c_2^6+4718592\sqrt{2}(8\gamma_2+3)^5(704\gamma_2(4\gamma_2+3)+459)c_2^5-11796480(8\gamma_2+$

$3)^6(22\gamma_2(4\gamma_2+3)+15)c_2^4+81920\sqrt{2}(8\gamma_2+3)^7(88\gamma_2(4\gamma_2+3)+63)c_2^3-384(8\gamma_2+3)^8(704\gamma_2(4\gamma_2+3)+531)c_2^2+48\sqrt{2}(8\gamma_2+$
$3)^9(64\gamma_2(4\gamma_2+3)+51)c_2-(8\gamma_2+3)^{10}(32\gamma_2(4\gamma_2+3)+27))c_1^6+944784(-61572651155456c_2^{13}+45079976738816\sqrt{2}(8\gamma_2+$
$3)c_2^{12}-103079215104(4672\gamma_2(4\gamma_2+3)+2661)c_2^{11}+34359738368\sqrt{2}(8\gamma_2+3)(2816\gamma_2(4\gamma_2+3)+1647)c_2^{10}-268435456(80\gamma_2(4\gamma_2+$
$3)(19360\gamma_2(4\gamma_2+3)+22707)+531927)c_2^9+1207959552\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(8096\gamma_2(4\gamma_2+3)+9813)+11853)c_2^8-12582912(8\gamma_2+$
$3)^2(16\gamma_2(4\gamma_2+3)(26048\gamma_2(4\gamma_2+3)+33129)+167265)c_2^7+352321536\sqrt{2}(8\gamma_2+3)^3(\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+963)+$
$324)c_2^6-589824(8\gamma_2+3)^4(8\gamma_2(4\gamma_2+3)(3344\gamma_2(4\gamma_2+3)+5169)+15363)c_2^5+4096\sqrt{2}(8\gamma_2+3)^5(80\gamma_2(4\gamma_2+3)(880\gamma_2(4\gamma_2+3)+$
$1773)+62613)c_2^4-128(8\gamma_2+3)^6(16\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+7407)+76707)c_2^3-384\sqrt{2}(8\gamma_2+3)^7(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)-3)-297)c_2^2+(8\gamma_2+3)^8(16\gamma_2(4\gamma_2+3)(544\gamma_2(4\gamma_2+3)+333)-1215)c_2-2\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^9(32\gamma_2(4\gamma_2+3)+$
$27))c_1^5-13122(-2251799813685248 0c_2^{14}+16008889300418560\sqrt{2}(8\gamma_2+3)c_2^{13}-2199023255552(74960\gamma_2(4\gamma_2+3)+43299)c_2^{12}+$
$137438953472\sqrt{2}(8\gamma_2+3)(229760\gamma_2(4\gamma_2+3)+141363)c_2^{11}-8589934592(16\gamma_2(4\gamma_2+3)(925760\gamma_2(4\gamma_2+3)+1159713)+5758533)c_2^{10}+$
$42949672960\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(31768\gamma_2(4\gamma_2+3)+44433)+120447)c_2^9-4026531840(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(78496\gamma_2(4\gamma_2+$
$3)+178281)+127332)+1867941)c_2^8+50331648\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(56320\gamma_2(4\gamma_2+3)+175203)+1180737)+$
$9630333)c_2^7-22020096(8\gamma_2+3)^2(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(3520\gamma_2(4\gamma_2+3)+29241)+251019)+2312631)c_2^6-131072\sqrt{2}(8\gamma_2+$
$3)^3(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(25520\gamma_2(4\gamma_2+3)-139509)-3235545)-16707951)c_2^5+40960(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+$
$3)(1298\gamma_2(4\gamma_2+3)-2781)-660231)-3692385)c_2^4-2560\sqrt{2}(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(3968\gamma_2(4\gamma_2+3)-8775)-$
$135351)-1565163)c_2^3+32(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(6080\gamma_2(4\gamma_2+3)-30753)-410427)-4721733)c_2^2+16\sqrt{2}(8\gamma_2+$
$3)^7(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(80\gamma_2(4\gamma_2+3)+1809)+9963)+111537)c_2-(8\gamma_2+3)^8(8\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)+$
$27)(40\gamma_2(4\gamma_2+3)+27))c_1^4+69984(-11258999068426240c_2^{15}+7740561859543040\sqrt{2}(8\gamma_2+3)c_2^{14}-21990232555520(3472\gamma_2(4\gamma_2+$
$3)+2061)c_2^{13}+549755813888\sqrt{2}(8\gamma_2+3)(25120\gamma_2(4\gamma_2+3)+17073)c_2^{12}-17179869184(176\gamma_2(4\gamma_2+3)(16960\gamma_2(4\gamma_2+3)+$
$24399)+1476873)c_2^{11}+2147483648\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(112640\gamma_2(4\gamma_2+3)+214281)+1386477)c_2^{10}-12884901888(5\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(10912\gamma_2(4\gamma_2+3)+41805)+301563)+396819)c_2^9+201326592\sqrt{2}(8\gamma_2+3)(20\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+$
$3)+9423)+338985)+1988469)c_2^8+6291456(8\gamma_2+3)^2(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(7040\gamma_2(4\gamma_2+3)-71199)-784161)-8070273)c_2^7-$
$262144\sqrt{2}(8\gamma_2+3)^3(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(12320\gamma_2(4\gamma_2+3)-68877)-871641)-9692055)c_2^6+16384(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(4840\gamma_2(4\gamma_2+3)-36261)-1972917)-11844063)c_2^5+2048\sqrt{2}(8\gamma_2+3)^5(80\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)+2781)+79461)+2645541)c_2^4-64(8\gamma_2+3)^6(160\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+2457)+40095)+3243321)c_2^3+$
$8\sqrt{2}(8\gamma_2+3)^7(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(640\gamma_2(4\gamma_2+3)+81)+10449)+303993)c_2^2-4(8\gamma_2+3)^8(4\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(160\gamma_2(4\gamma_2+3)-621)-2673)+6561)c_2-27\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^9(32\gamma_2(4\gamma_2+3)+27))c_1^3-81(-144115188075855872 00c_2^{16}+$
$95476312100254515 20\sqrt{2}(8\gamma_2+3)c_2^{15}-6755399441055744(13280\gamma_2(4\gamma_2+3)+8241)c_2^{14}+70368744177664\sqrt{2}(8\gamma_2+3)(215600\gamma_2(4\gamma_2+$
$3)+171459)c_2^{13}-10995116277760(32\gamma_2(4\gamma_2+3)(144632\gamma_2(4\gamma_2+3)+261405)+3275559)c_2^{12}+824633720832\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(242720\gamma_2(4\gamma_2+3)+755493)+5864859)c_2^{11}-8589934592(32\gamma_2(4\gamma_2+3)(2920\gamma_2(4\gamma_2+3)(3520\gamma_2(4\gamma_2+3)+36603)+118670589)+$
$1113354315)c_2^{10}-1073741824\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(344960\gamma_2(4\gamma_2+3)-6820533)-75439107)-793854027)c_2^9+$
$603979776(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(149600\gamma_2(4\gamma_2+3)-1142751)-44930457)-507514491)-1822681521)c_2^8-$
$2147483648\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(40\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(1078\gamma_2(4\gamma_2+3)-9333)-925587)-29953881)-29596671)c_2^7-$
$524288(128\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(24640\gamma_2(4\gamma_2+3)+940671)+21925485)+992114325)+$
$4274484939)+102025393641)c_2^6+196608\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(20\gamma_2(4\gamma_2+3)(3616\gamma_2(4\gamma_2+$
$3)-6009)+1133487)+39062007)+793170225)+10379849733)c_2^5-4096(8\gamma_2+3)^2(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(128\gamma_2(4\gamma_2+3)(2645\gamma_2(4\gamma_2+3)-26136)+1629477)+179334729)+4360250331)+30926854701)c_2^4-2048\sqrt{2}(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(800\gamma_2(4\gamma_2+3)+115497)+33939)-30192993)-409347351)-1516201173)c_2^3+$
$384(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(704\gamma_2(4\gamma_2+3)+8331)-351)-11982573)-39503781)-$
$291761109)c_2^2-16\sqrt{2}(8\gamma_2+3)^5(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(1280\gamma_2(4\gamma_2+3)+7227)-15795)-$
$1948617)-22851963)-81310473)c_2+(8\gamma_2+3)^6(32\gamma_2(4\gamma_2+3)+27)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(320\gamma_2^2(4\gamma_2+3)^2-6561)-59049)-$
$531441))c_1^2+144(-634106827533765836 8c_2^{17}+4035225266123964416\sqrt{2}(8\gamma_2+3)c_2^{16}-4503599627370496(7976\gamma_2(4\gamma_2+3)+$
$5283)c_2^{15}+140737488355328\sqrt{2}(8\gamma_2+3)(39920\gamma_2(4\gamma_2+3)+39411)c_2^{14}-2199023255552(32\gamma_2(4\gamma_2+3)(235480\gamma_2(4\gamma_2+3)+589131)+$
$8438823)c_2^{13}+274877906944\sqrt{2}(8\gamma_2+3)(256\gamma_2(4\gamma_2+3)(11147\gamma_2(4\gamma_2+3)+69507)+10267641)c_2^{12}+8589934592(32\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(29504\gamma_2(4\gamma_2+3)-6545907)-70427961)-737840583)c_2^{11}-2147483648\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(149248\gamma_2(4\gamma_2+3)-1696977)-24709455)-304505487)c_2^{10}+67108864(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(487520\gamma_2(4\gamma_2+$
$3)-4492647)-267871779)-3708547659)-15154801191)c_2^9-33554432\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+$
$3)(19360\gamma_2(4\gamma_2+3)-696699)-56124009)-1932030063)-2203465923)c_2^8-524288(512\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+$
$3)(2\gamma_2(4\gamma_2+3)(88000\gamma_2(4\gamma_2+3)-423261)+9452133)+1213136919)+1493145819)+153420226065)c_2^7+131072\sqrt{2}(8\gamma_2+$
$3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(160\gamma_2(4\gamma_2+3)(1232\gamma_2(4\gamma_2+3)-16839)+11239641)+430173423)+4499461629)+$
$29985731739)c_2^6+4096(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(40\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(496\gamma_2(4\gamma_2+3)+70011)-1175877)-$
$156493701)-19858237749)-266410133271)-656224941123)c_2^5-1024\sqrt{2}(8\gamma_2+3)(8\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(5360\gamma_2(4\gamma_2+3)+22491)-6841665)-146805291)-4481641953)-59743357191)-73834692453)c_2^4+$
$128(8\gamma_2+3)^2(128\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(5440\gamma_2(4\gamma_2+3)-89019)-11718675)-$

$197419761)-2836576179)-1163324349)-22972600107)c_2^3+16\sqrt{2}(8\gamma_2+3)^3(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+3)+12555)+771525)+5181003)+69264477)+1784047437)+2195382771)c_2^2-(8\gamma_2+3)^4(16\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(608\gamma_2(4\gamma_2+3)+10287)+406539)+518319)+24977727)+157837977)+$
$387420489)c_2+256\sqrt{2}\gamma_2^3(4\gamma_2+3)^3(8\gamma_2+3)^5(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729))c_1+64(4611686018427387904c_2^{18}-$
$281024616747918950 4\sqrt{2}(8\gamma_2+3)c_2^{17}+13510798882111488(1744\gamma_2(4\gamma_2+3)+1263)c_2^{16}-20266198323167232\sqrt{2}(8\gamma_2+3)(166\gamma_2(4\gamma_2+$
$3)+215)c_2^{15}+52776558133248(16\gamma_2(4\gamma_2+3)(9920\gamma_2(4\gamma_2+3)+37989)+308529)c_2^{14}-824633720832\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(8456\gamma_2(4\gamma_2+3)+151845)+3372381)c_2^{13}-51539607552(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(45616\gamma_2(4\gamma_2+3)-861873)-23305671)-$
$138661875)c_2^{12}+3221225472\sqrt{2}(8\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(73984\gamma_2(4\gamma_2+3)-809253)-18279837)-268027785)c_2^{11}-$
$201326592(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(54208\gamma_2(4\gamma_2+3)-704625)-102240387)-1730283687)-7827786945)c_2^{10}-$
$16777216\sqrt{2}(8\gamma_2+3)(64\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(\gamma_2(4\gamma_2+3)(36080\gamma_2(4\gamma_2+3)-230949)+4718331)+391378959)+7796915253)c_2^9+$
$25165824(2\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(51040\gamma_2(4\gamma_2+3)-694773)+19038645)+2764517445)+$
$14576882067)+6169498569)c_2^8-196608\sqrt{2}(8\gamma_2+3)(128\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(10\gamma_2(4\gamma_2+3)(256\gamma_2(4\gamma_2+$
$3)-21633)+1508571)+122552919)+1397464569)+40192784415)c_2^7-36864(16\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+$
$3)(2\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(2240\gamma_2(4\gamma_2+3)+5573)-3293451)-23760945)-880465059)-53802187983)-147097377243)c_2^6+$
$49152\sqrt{2}(8\gamma_2+3)(4\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(128\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(680\gamma_2(4\gamma_2+3)-14049)-399411)-$
$7680015)-1704147579)-3962955537)-3030808023)c_2^5+384(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(1792\gamma_2(4\gamma_2+3)(5\gamma_2(4\gamma_2+3)+276)-2337255)-70005141)-147515337)+19515753549)+47397628467)+130130237583)c_2^4-$
$48\sqrt{2}(8\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+3)(4\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(512\gamma_2(4\gamma_2+3)+2589)-$
$1006749)-66568149)-179423667)-524886561)+2319739965)+11751754833)c_2^3+3(8\gamma_2+3)^2(256\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(2\gamma_2(4\gamma_2+3)(32\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)(64\gamma_2(4\gamma_2+3)-6183)-770391)-48208041)-70996581)-336402153)-$
$43046721)+1937102445)c_2^2+12\sqrt{2}\gamma_2(4\gamma_2+3)(8\gamma_2+3)^3(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+81)+729)(16\gamma_2(4\gamma_2+3)(16\gamma_2(4\gamma_2+$
$3)(16\gamma_2(4\gamma_2+3)(8\gamma_2(4\gamma_2+3)+195)+4617)+32805)+59049)c_2-2\gamma_2^2(4\gamma_2+3)^2(8\gamma_2+3)^4(32\gamma_2(4\gamma_2+3)+27)(16\gamma_2(4\gamma_2+$
$3)(8\gamma_2(4\gamma_2+3)+81)+729)^2))$.

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