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INTEGRABILITY OF PLANAR POLYNOMIAL DIFFERENTIAL SYSTEMS THROUGH LINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT. In this work we consider rational ordinary differential equations $dy/dx = Q(x,y)/P(x,y)$, with $Q(x,y)$ and $P(x,y)$ coprime polynomials with real coefficients. We give a method to construct equations of this type for which a first integral can be expressed from two independent solutions of a second-order homogeneous linear differential equation. This first integral is, in general, given by a non Liouvillian function.

We show that all the known families of quadratic systems with an irreducible invariant algebraic curve of arbitrarily high degree and without a rational first integral, can be constructed by using this method. We also present a new example of this kind of family.

We give an analogous method for constructing rational equations but by means of a linear differential equation of first order.

1. Introduction. This paper deals with rational ordinary differential equations such as

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)}, \tag{1}$$

where $Q(x,y)$ and $P(x,y)$ are coprime polynomials with real coefficients. We associate to this rational equation a planar polynomial differential system by introducing an independent variable t usually called *time*. Denoting by $\dot{} = d/dt$, we have

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{2}$$

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