

The problem of distinguishing between a center and a focus for nilpotent and degenerate analytic systems

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Received 8 June 2005; revised 21 December 2005

Available online 24 April 2006

We dedicate this paper to the memory of Javier Chavarriga

Abstract

In this work we study the centers of planar analytic vector fields which are limit of linear type centers. It is proved that all the nilpotent centers are limit of linear type centers and consequently the Poincaré–Liapunov method to find linear type centers can be also used to find the nilpotent centers. Moreover, we show that the degenerate centers which are limit of linear type centers are also detectable with the Poincaré–Liapunov method.

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MSC: primary 34C05; secondary 58F14

Keywords: Nilpotent center; Degenerate center; Liapunov constants; Cyclicity

1. Introduction and statement of the main results

Two of the main and oldest problems in the qualitative theory of differential systems in $\mathbb{R}^2$ are the distinction between a center and a focus, called the *center problem*; and the determination

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