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On the shape of limit cycles that bifurcate from Hamiltonian centers

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1. Introduction

The main problem in the qualitative theory of real planar differential equations is the determination of limit cycles. Limit cycles of planar vector fields were defined by Poincaré [13]. At the end of the 1920s van der Pol [14], Liénard [11] and Andronov [1] proved that a closed trajectory of a self-sustained oscillation occurring in a vacuum tube circuit was a limit cycle as considered by Poincaré. After these works, the non-existence, existence, uniqueness and other properties of limit cycles were studied extensively by mathematicians and physicists, and more recently also by chemists, biologists, economists, etc. (see for instance, the books [5, 17]).

In 1881–1886 Poincaré defined the notion of a center on the plane, as an isolated singular point surrounded by periodic orbits. Then one way to produce limit cycles is by perturbing a system which has a center, in such a way that limit cycles bifurcate in the perturbed system from some of the periodic orbits in the original system [15]. An understanding of this problem gives us an idea of how vast the second part of Hilbert's 16th problem really is [9], i.e. what is the maximum number of limit cycles of polynomial differential systems of a given degree? This problem is still unsolved even for the quadratic polynomial differential systems.

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