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SEMISTABLE LIMIT CYCLES THAT BIFURCATE FROM CENTERS

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Suppose that the differential system

$$\dot{x} = P_0(x,\, y) + \sum_{i,j=1}^{m} a_{ij}(\varepsilon) x^i y^j\,,$$

$$\dot{y} = Q_0(x,\, y) + \sum_{i,j=1}^{m} b_{ij}(\varepsilon) x^i y^j$$

has a center at the origin for $\varepsilon = 0$, where P_0, Q_0, a_{ij} and b_{ij} are analytic functions in their variables, such that $a_{ij}(0) = b_{ij}(0) = 0$. We present an analytic method to compute the semistable limit cycles which bifurcate from the periodic orbits of the analytic center, up to an arbitrary order in the perturbation parameter ε. We also provide an algorithm for the computation of the saddle–node bifurcation hypersurface of limit cycles in the parameter space $\{a_{ij}, b_{ij}\}_{1\le i,j\le m}$. As an example, we apply the method to compute, first, the analytic expression of the unique semistable limit cycle of the Liénard system

$$\begin{aligned}\dot{x} &= y + \varepsilon(a_1 x + a_3 x^3 + a_5 x^5)\\ &= y + \sum_{k=1}^{\infty} \varepsilon^k (a_{1k} x + a_{3k} x^3 + a_{5k} x^5)\,,\\ \dot{y} &= -x\,,\end{aligned}$$

and second, an approximation of the saddle-node bifurcation surface of limit cycles in the parameter space $(a_1,\, a_3,\, a_5)$. Both computations are valid for ε sufficiently small.

Keywords: Limit cycle-bifurcation; center-planar vector field.