ORIGINAL ARTICLE

Symmetric planar central configurations of five bodies: Euler plus two

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Abstract We study planar central configurations of the five-body problem where three of the bodies are collinear, forming an Euler central configuration of the three-body problem, and the two other bodies together with the collinear configuration are in the same plane. The problem considered here assumes certain symmetries. From the three bodies in the collinear configuration, the two bodies at the extremities have equal masses and the third one is at the middle point between the two. The fourth and fifth bodies are placed in a symmetric way: either with respect to the line containing the three bodies, or with respect to the middle body in the collinear configuration, or with respect to the perpendicular bisector of the segment containing the three bodies. The possible stacked five-body central configurations satisfying these types of symmetries are: a rhombus with four masses at the vertices and a fifth mass in the center, and a trapezoid with four masses at the vertices and a fifth mass at the midpoint of one of the parallel sides.

Keywords Central configuration · Planar five-body problem · Stacked central configurations

1 Introduction

Let $(m_1, m_2, ..., m_n)$ be *n* positive masses in the plane, of position vectors $(r_1, r_2, ..., r_n)$, respectively, subject to Newtonian gravitation. The motion of the system is governed by the equations

$$m_i \ddot{r}_i = \frac{\partial U}{\partial r_i}, \quad i = 1, \dots, n,$$

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