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The Center Problem for a Linear Center Perturbed by Homogeneous Polynomials

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Abstract The centers of the polynomial differential systems with a linear center perturbed by homogeneous polynomials have been studied for the degrees $s = 2, 3, 4, 5$. They are completely classified for $s = 2, 3$, and partially classified for $s = 4, 5$. In this paper we recall these results for $s = 2, 3, 4, 5$, and we give new centers for $s = 6, 7$

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1 Introduction

One of the classical problems in the qualitative theory of planar analytic differential systems is to characterize the local phase portrait at an isolated singular point. This problem has been solved unless if the singular point is of focus-center type, see [1, 2, 3]. Recall that a singular point is said to be of focus-center type if it is either a focus or a center. The problem of distinguishing between a center or a focus is called the *center problem*. Of course, if the linear part of the singular point is non-degenerate (i.e. its determinant does not vanish) the characterization is well known, see [4, 5]. If an analytic system has a non-degenerate center at the origin, then after a linear change of variables and a rescaling of the time variable it can be written in the form:

$$\dot{x} = -y + X(x, y), \qquad \dot{y} = x + Y(x, y), \tag{1}$$

where $X(x, y)$ and $Y(x, y)$ are analytic functions without constant and linear terms defined in a certain neighborhood of the origin.

The aim of this paper consists in giving a characterization of the center cases for some planar polynomial differential systems with a linear center perturbed by homogeneous polynomials i.e., systems of the form (1) with $X(x, y) \equiv X_s(x, y)$ and $Y(x, y) \equiv Y_s(x, y)$, where $X_s(x, y)$ and $Y_s(x, y)$ are homogeneous polynomials of degree s, with $s \geq 2$. The centers of the polynomial differential systems (1) with homogeneous polynomial perturbations have been studied for the degrees $s = 2, 3, 4, 5$. They are completely classified for $s = 2, 3$, see for instance Kapteyn [6, 7], Malkin [8] and Sibirskii [9], and partially classified for $s = 4, 5$, see Chavarriga and the author [10, 11]. In Section 3 we recall these results for $s = 2, 3, 4, 5$, and we give new centers for $s = 6, 7$.

Therefore, we consider the two-dimensional autonomous differential system

$$\dot{x} = -y + X_s(x, y), \qquad \dot{y} = x + Y_s(x, y), \tag{2}$$

where $X_s(x, y)$ and $Y_s(x, y)$ are homogeneous polynomials of degree s, with $s \geq 2$.

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