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Lower bounds for the local cyclicity for families of centers

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Abstract

In this paper, we are interested in how the local cyclicity of a family of centers depends on the parameters. This fact was pointed out in [21], to prove that there exists a family of cubic centers, labeled by CD_{31}^{12} in [25], with more local cyclicity than expected. In this family, there is a special center such that at least twelve limit cycles of small amplitude bifurcate from the origin when we perturb it in the cubic polynomial general class. The original proof has some crucial missing points in the arguments that we correct here. We take advantage of a better understanding of the bifurcation phenomenon in nongeneric cases to show two new cubic systems exhibiting 11 limit cycles and another exhibiting 12. Finally, using the same techniques, we study the local cyclicity of holomorphic quartic centers, proving that 21 limit cycles of small amplitude bifurcate from the origin, when we perturb in the class of quartic polynomial vector fields.

MSC: 34C23; 34C25; 34C07

Keywords: Small-amplitude limit cycle; Polynomial vector field; Center cyclicity; Lyapunov constants; Higher-order developments and parallelization

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