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Linearizability and integrability of vector fields via commutation ☆

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Abstract

In this paper, we consider complex smooth and analytic vector fields $\mathcal{X}$ in a neighborhood of a nondegenerate singular point. It is proved the equivalence between linearizability and commutation, i.e., the existence of a commuting vector field $\mathcal{Y}$ such that the Lie brackets $[\mathcal{X}, \mathcal{Y}] \equiv 0$. For complex smooth and analytic vector fields in the plane and in a neighborhood of a nondegenerate singular point, it is also proved the equivalence between integrability and the existence of a smooth vector field $\mathcal{Y}$, such that $\mathcal{Y}$ is a normalizer of $\mathcal{X}$, i.e., $[\mathcal{X}, \mathcal{Y}] = \mu\mathcal{X}$.

Keywords: Polynomial vector fields; Linearizability; Integrability; Commutation

1. Introduction

This work was originated from the interest of the authors in isochronous foci and isochronous centers of vector fields in the plane, see, for instance, [7,8]. An isochronous focus (respectively, center) is a singular point of a vector field of focus (respectively, center) type for which there exists a section Σ, that is a simple arc without contact with the singular point as an endpoint,

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