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Coexistence of algebraic and non-algebraic limit cycles, explicitly given, using Riccati equations

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Abstract

We give a family of planar polynomial differential systems whose limit cycles can be explicitly described using polar coordinates. Moreover, we characterize the multiplicity of each one of the limit cycles whenever they exist. The given family of planar polynomial differential systems can have at most two limit cycles, counted with multiplicity.

As an application of this result we give an example of a planar polynomial differential system with two explicit limit cycles, one of them algebraic and the other one non-algebraic.

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1. Introduction and statement of the results

Our purpose in this work is to give a family of planar polynomial differential systems of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are coprime polynomials, for which an explicit expression of its limit cycles can be given. A *limit cycle* of system (1) is an isolated periodic orbit and it is said to be *algebraic* if it is contained in the zero set of an algebraic curve. We give an example of a system of form (1) with two limit cycles: one of them is algebraic and the other one is shown to be non-algebraic.

An important problem of the qualitative theory of differential equations is to determine the limit cycles of a system of form (1). We usually only ask for the number of such limit cycles, but their location as orbits of the system is also an interesting problem. And an even more difficult problem is to give an explicit expression of them. We are able to solve this last problem for a given family of systems of form (1). Until recently, the only limit cycles known

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