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ON THE POLYNOMIAL LIMIT CYCLES OF POLYNOMIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

In this paper we deal with ordinary differential equations of the form dy/dx = P(x, y) where P(x, y) is a real polynomial in the variables x and y, of degree n in the variable y. If $y = \varphi(x)$ is a solution of this equation defined for $x \in [0, 1]$ and which satisfies $\varphi(0) = \varphi(1)$, we say that it is a periodic orbit. A limit cycle is an isolated periodic orbit in the set of all periodic orbits. If $\varphi(x)$ is a polynomial, then $\varphi(x)$ is called a polynomial solution.

We study the maximum number and the multiplicity of the polynomial limit cycles of dy/dx = P(x, y) with respect to n. We prove that this differential equation has at most n polynomial limit cycles and that this bound is sharp. If n = 1 (linear equation) or n = 2 (Riccati equation), we prove that the differential equation dy/dx = P(x, y) has at most npolynomial limit cycles counted with their multiplicities. For n = 3 (Abel equation), we show that at most three polynomial limit cycles can exist whereas the multiplicity of a polynomial limit cycle can be unbounded.

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